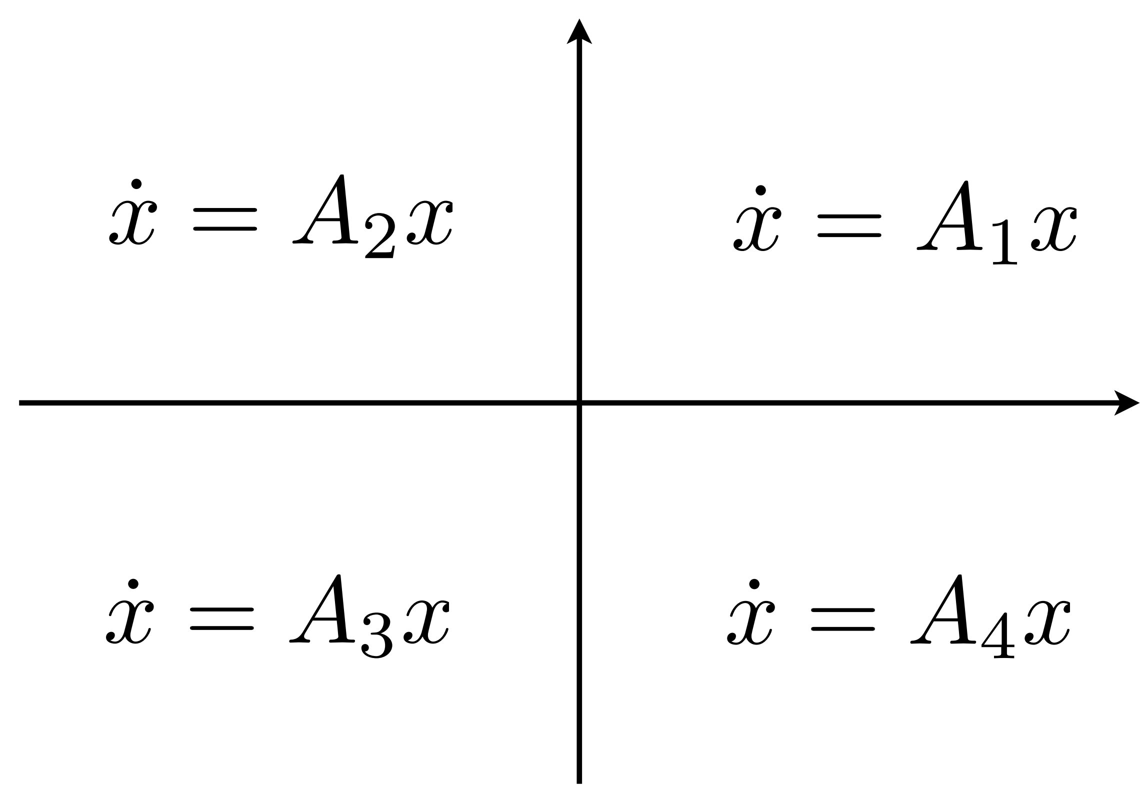
Lecture 7

Review

- Switched systems (subclass of hybrid systems)
- Review of continuous systems
- Stability properties of switched systems
  - Multiple Lyapunov functions
  - Common Lyapunov function

Review (stability of switched systems)

$$
\begin{aligned}
Q &= \{1, 2, \ldots, m\} \\
X &= \mathbb{R}^n \\
D(q) &= \Omega_q \\
(q, q') &\in E \text{ if } D^c(q) \cap D^c(q') \neq \emptyset \\
G(q, q') &= D^c(q) \cap D^c(q') \\
R(q, q', x) &= \{x\}
\end{aligned}
$$



Review (multiple Lyapunov functions)

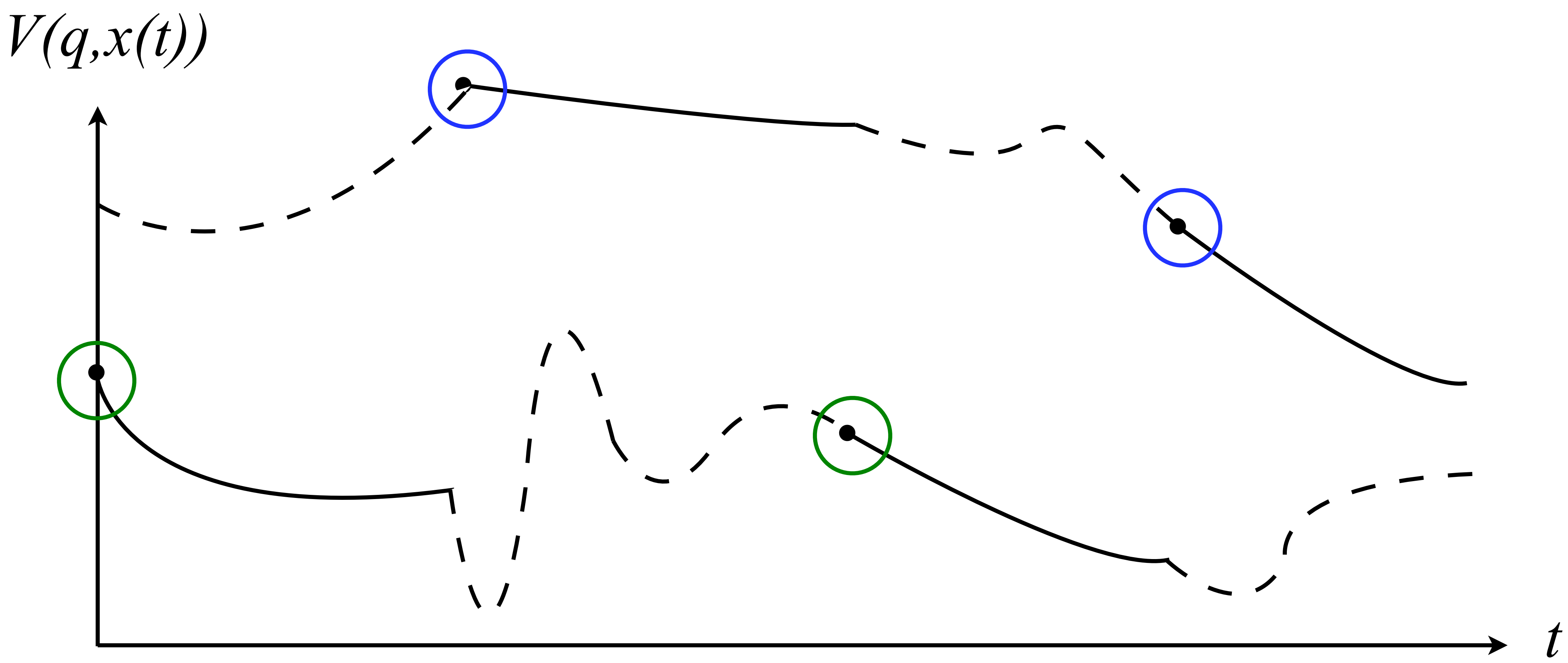
Suposse $x^* = 0$ is an equilibrium point of each mode $q = 1, \ldots, m$ of

$$\dot{x} = f_q(x), x \in \Omega_q \tag{1}$$

If there exist functions $V_1, \ldots V_m$ such that

$$\begin{aligned} V_q(0) &= 0 && (2) \\ V_q(x) &> 0 \;\; \forall x \in \mathbb{R}^n && (3) \\ V_q(x(t)) &\leq 0 \;\; \text{whenever} \;\; x(t) \in \Omega_q && (4) \end{aligned}$$

AND the sequence $\{V_q(x(\tau_{i,q}))\}$ is non-decreasing, where $\tau_{i,q}$ are the time instances when the mode q becomes active, then x^* is stable in the sense of Lyapunov



V(q,x(t))
t

Review (common Lyapunov function)

If there exist ε, $P > 0$ such that

$$PA_q + A_q^\top P < -\varepsilon I, \quad q = 1, \ldots, m$$

Then the origin is asymptotically stable.

Review (stabilizing sequence)

Suppose there exist $\mu_q \geq 0$, $q \in Q$ and

$$\sum_{q=1}^{m} \mu_q = 1 \tag{1}$$

such that

$$A = \sum_{q=1}^{m} \mu_q A_q \tag{2}$$

is stable.
Then a stabilizing sequence

$$\sigma : [0, \infty) \to Q = \{1, \ldots, m\} \tag{3}$$

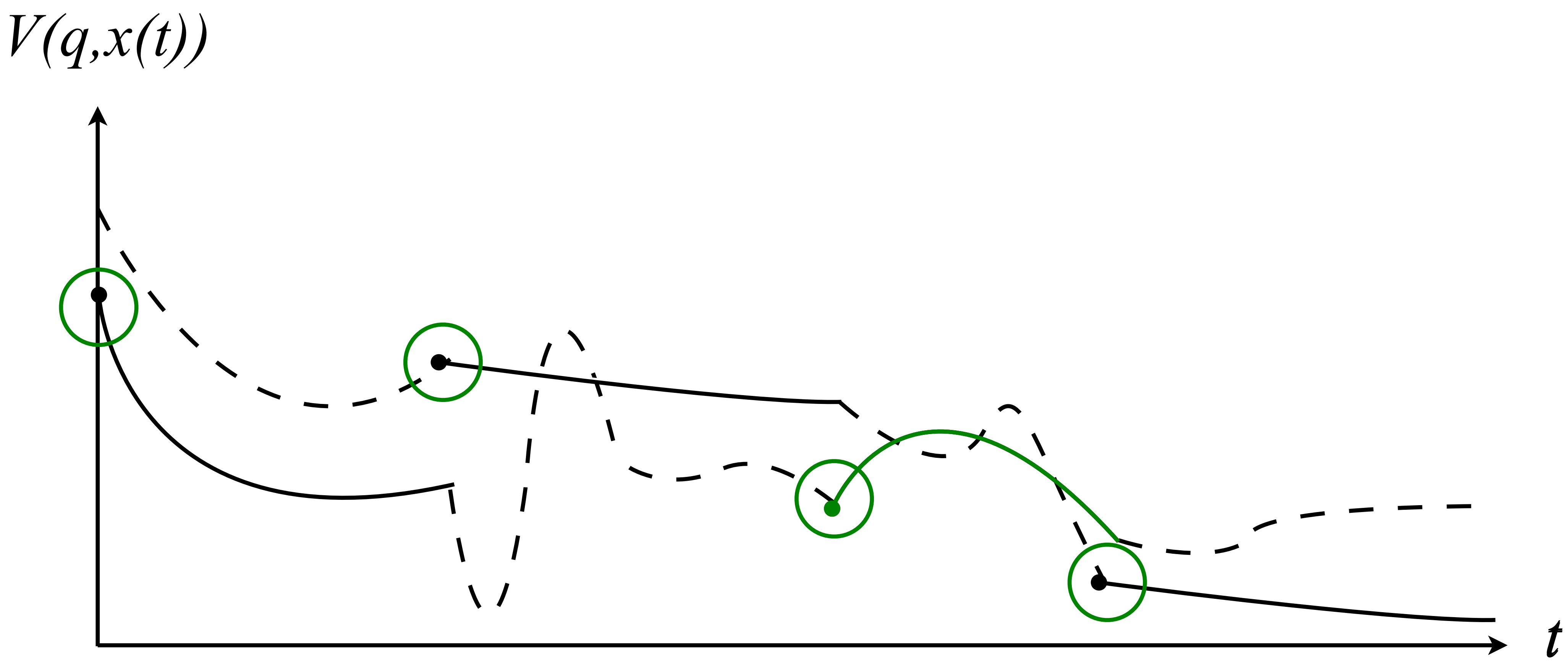
for $\dot{x} = A_\sigma x$ is given by

$$\sigma(x) = argmin_{q \in Q} \left\{ x^\top (A_q^\top P + P A_q) x \right\} \tag{4}$$

where P is the solution to $A^\top P + PA = -I$

Today

- Problems
- Stability of Hybrid Systems (Theorem 5)
- Next class:
  - Presentations (15 min)
    - Motivation
    - Present your model
    - Useful application of HS?
    - How would you identify/validate your model?
- Identification of hybrid systems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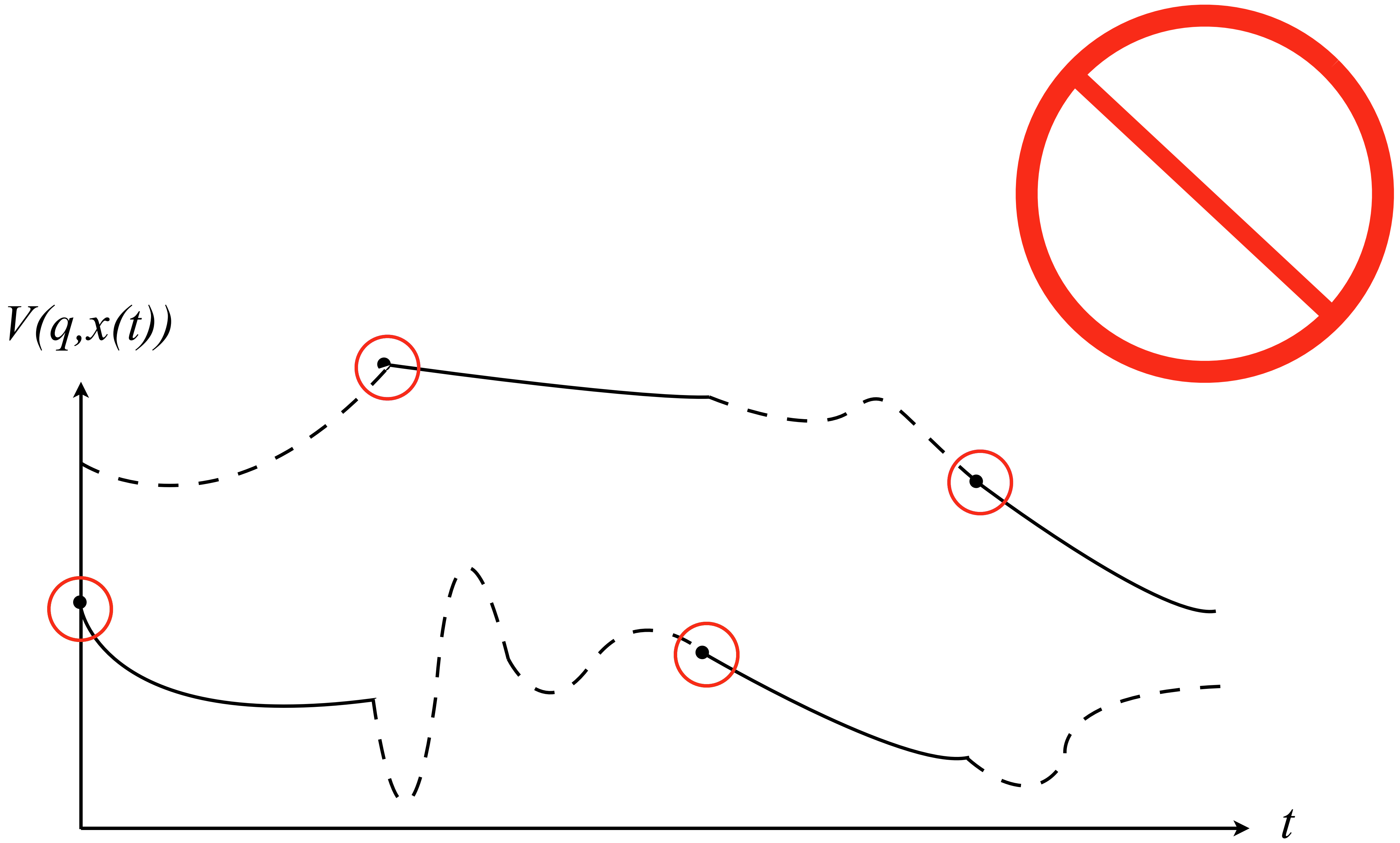
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