Lecture 12

Review

- Discrete Kalman Filter
- State-space observer (no-noise case)
- Simulations

Today

- Identification of hybrid system

Review (LTI system)

Consider the system

$$x_{k+1} = Ax_k + Bu_k + v_k$$
$$y_k = Cx_k + Du_k + w_k$$

Given only the input u_k and the output y_k over a period $k \in \{1,...,T\}$, determine:

1. The unknown order and relative degree
2. All system parameters $(A,\ B,\ C,\ D)$ and states $\{x_k\}$ for $k \in \{0,...,T\}$

Identification of hybrid systems

Consider the system PieceWise Affine (PWA) model

$$
\begin{aligned}
x_{k+1} &= A_{\sigma(k)} x_k + B_{\sigma(k)} u_k + f_{\sigma(k)} + v_k \\
y_k &= C_{\sigma(k)} x_k + D_{\sigma(k)} u_k + g_{\sigma(k)} + w_k
\end{aligned}
$$

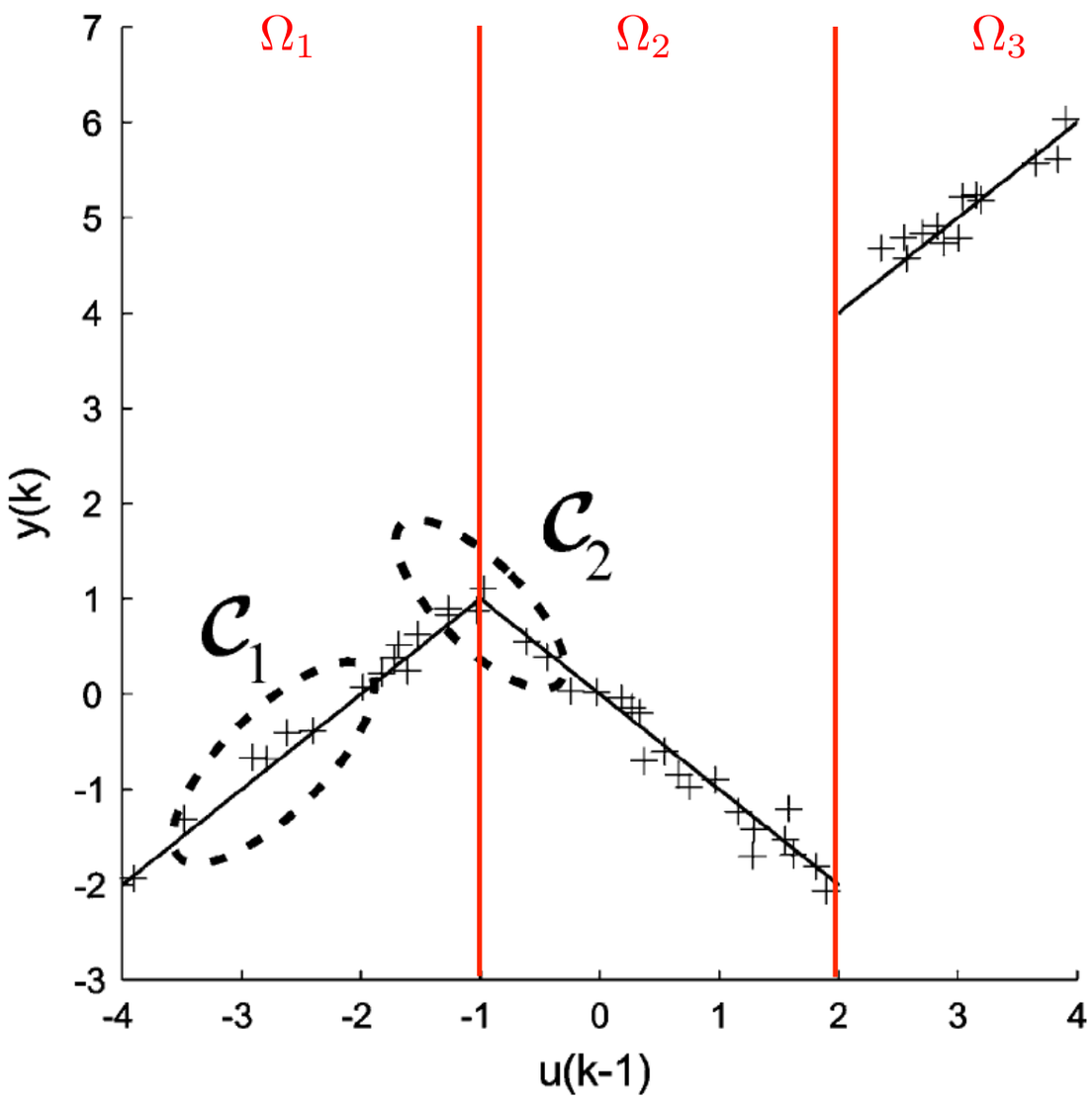
where is $\sigma(k)$ is given by the rule $\sigma(k) = i$ if and only if the discrete state $i = 1,\ldots,s$ is "active" at time k

Identification of hybrid systems

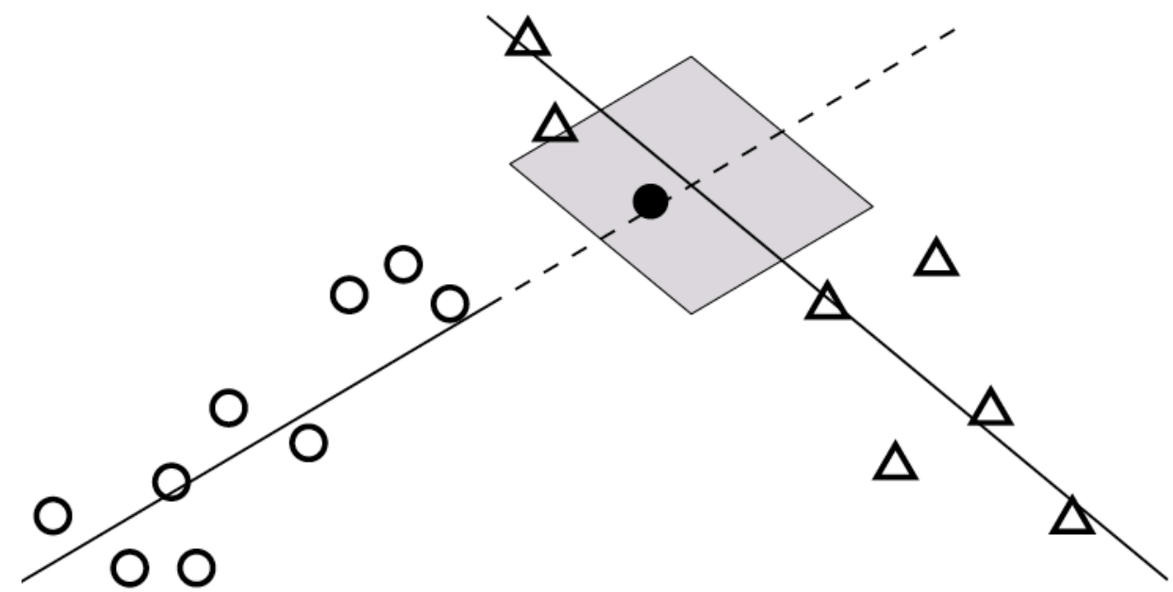
Given only the input u_k and the output y_k over a period $k \in \{0,\ldots,T\}$, determine:

1. The times where the discrete switching state changes values, i.e., the piecewise-constant function $\sigma(k)$
2. The number of discrete states s, i.e, the number of LTI systems involved
3. The unknown order and relative degree
4. All system parameters $(A_i,\ B_i,\ f_i,\ C_i,\ D_i,\ g_i)$ and states $\{x_k\}$ for $k \in \{0,\ldots,T\}$ and $i \in \{1,\ldots,s\}$

Example

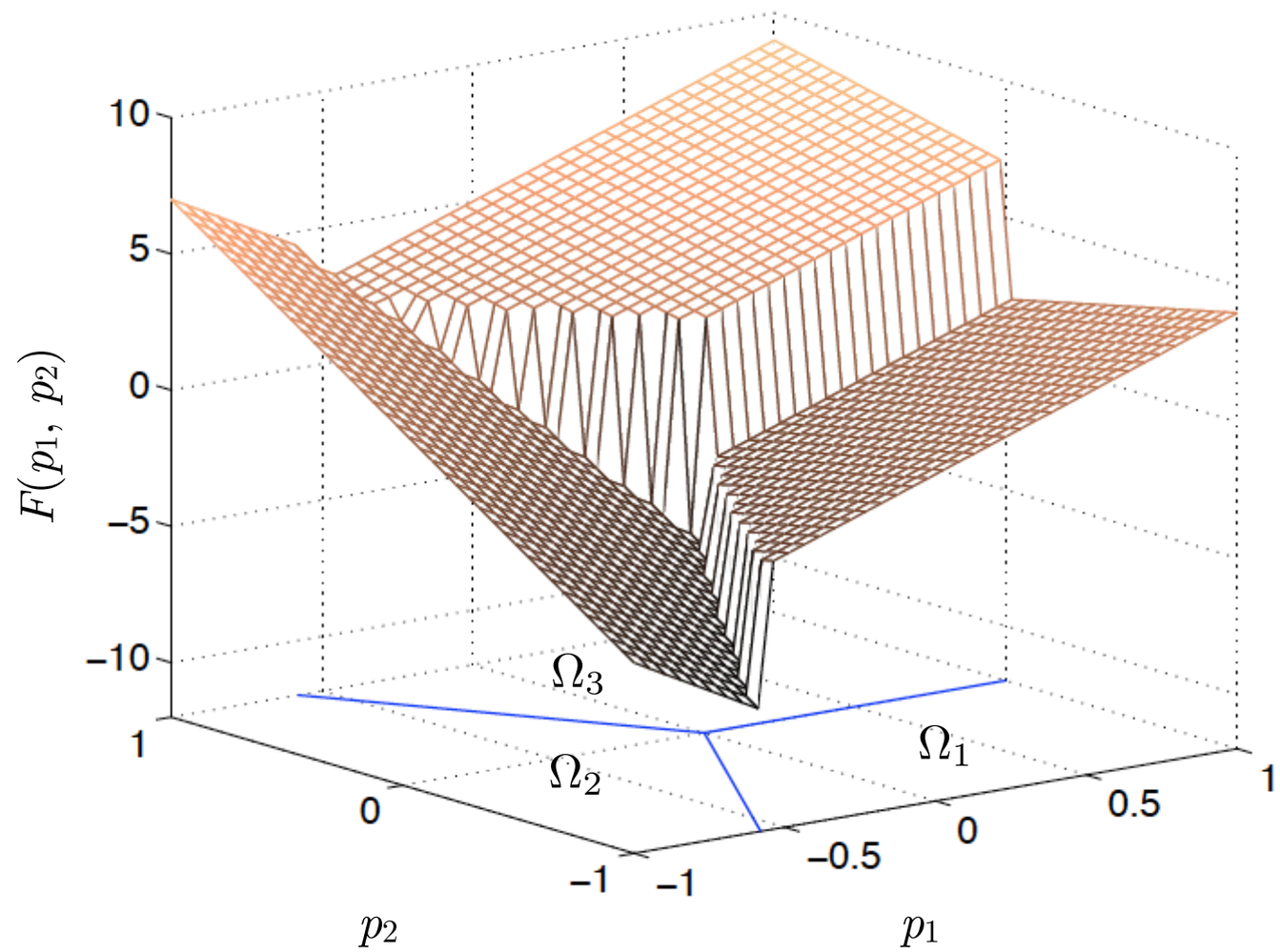


$$|y_k - h_k^\top \theta_i| < \delta \ \text{ for both } \ i = 1, 2$$



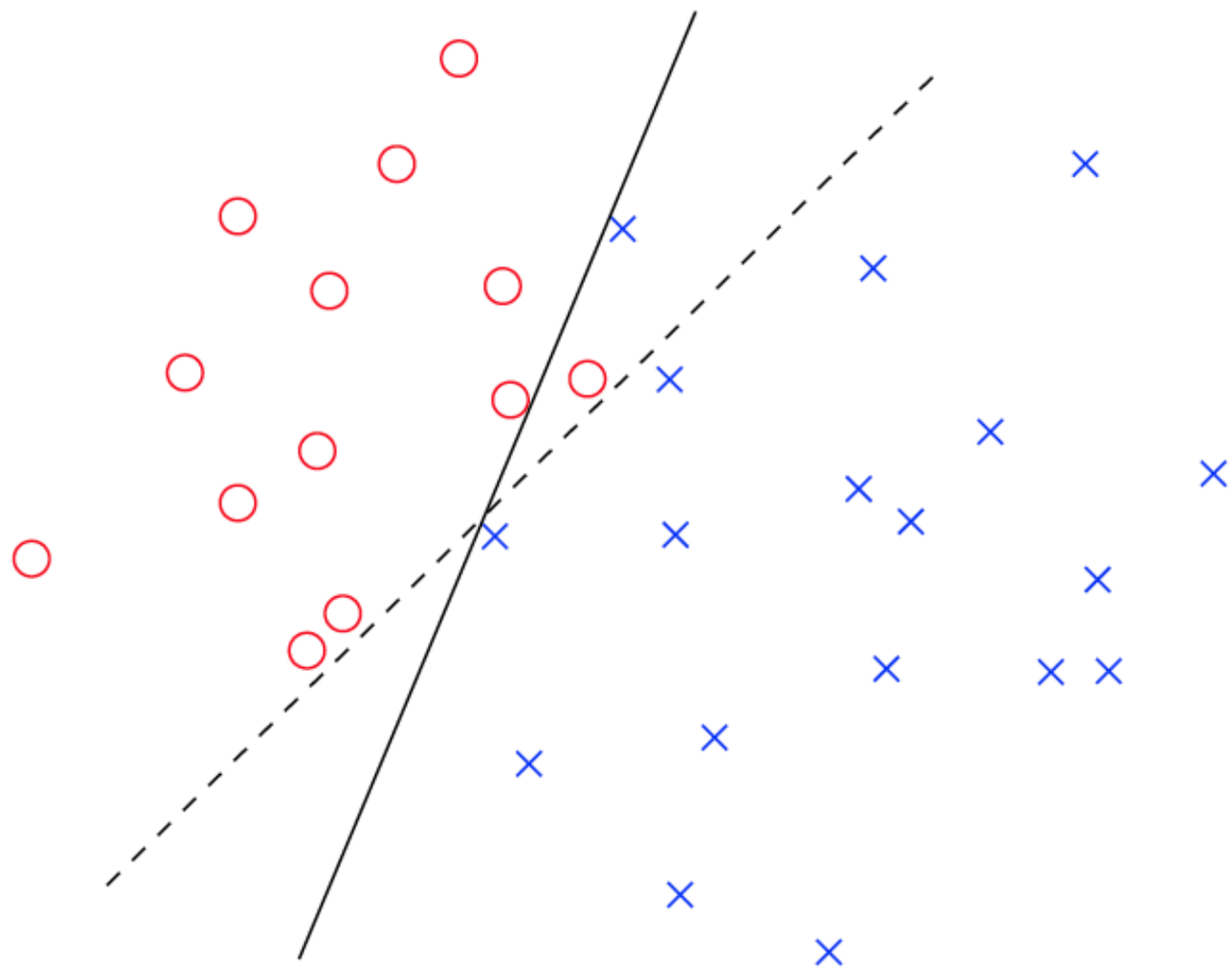
PWA Model

- Classification problem
- Let $\{\Omega_i\}_{i=1}^{s}$ be a complete partition of the domain
- Possible issues?
- Data samples
- Region estimation

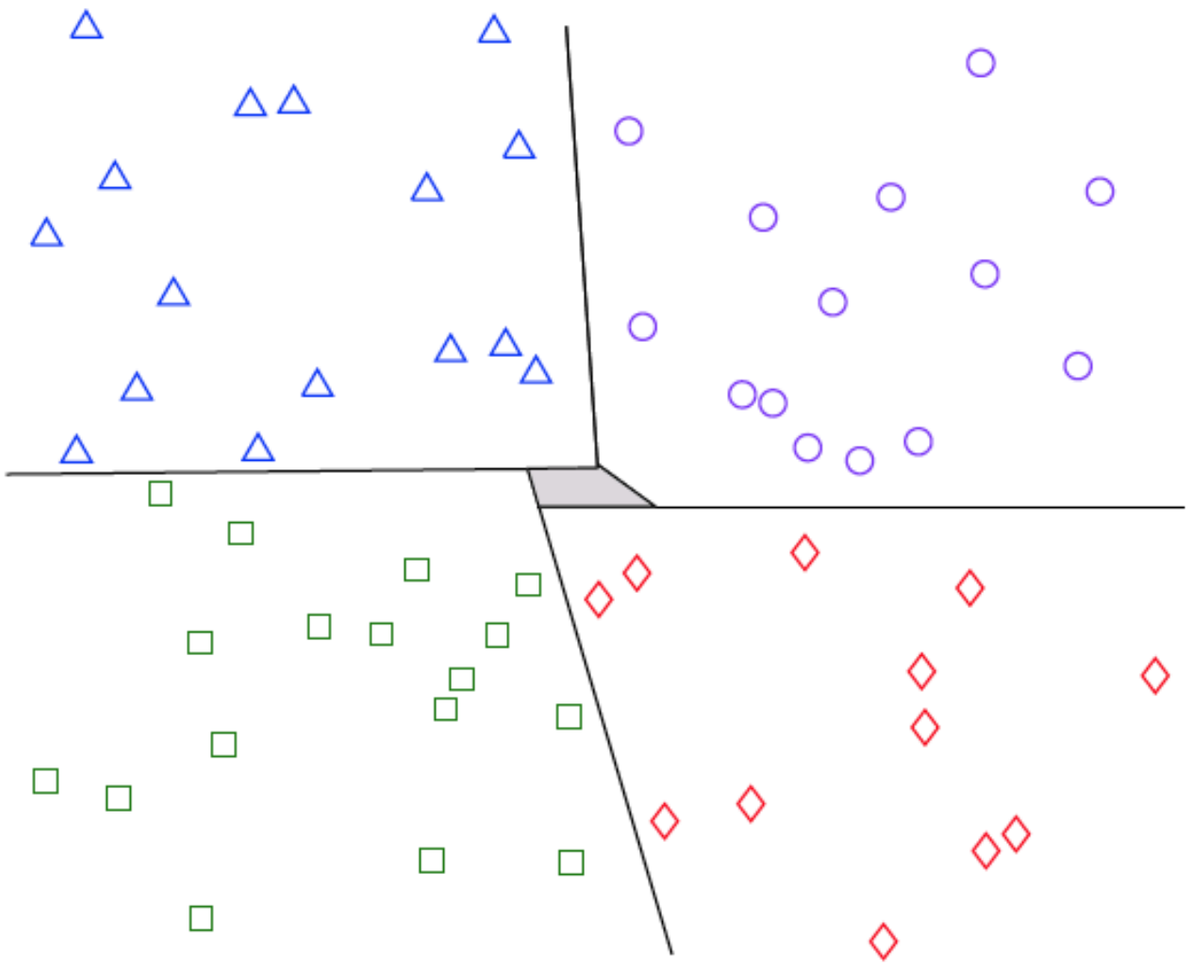


Discontinuous PWA of with parameters and $s = 3$

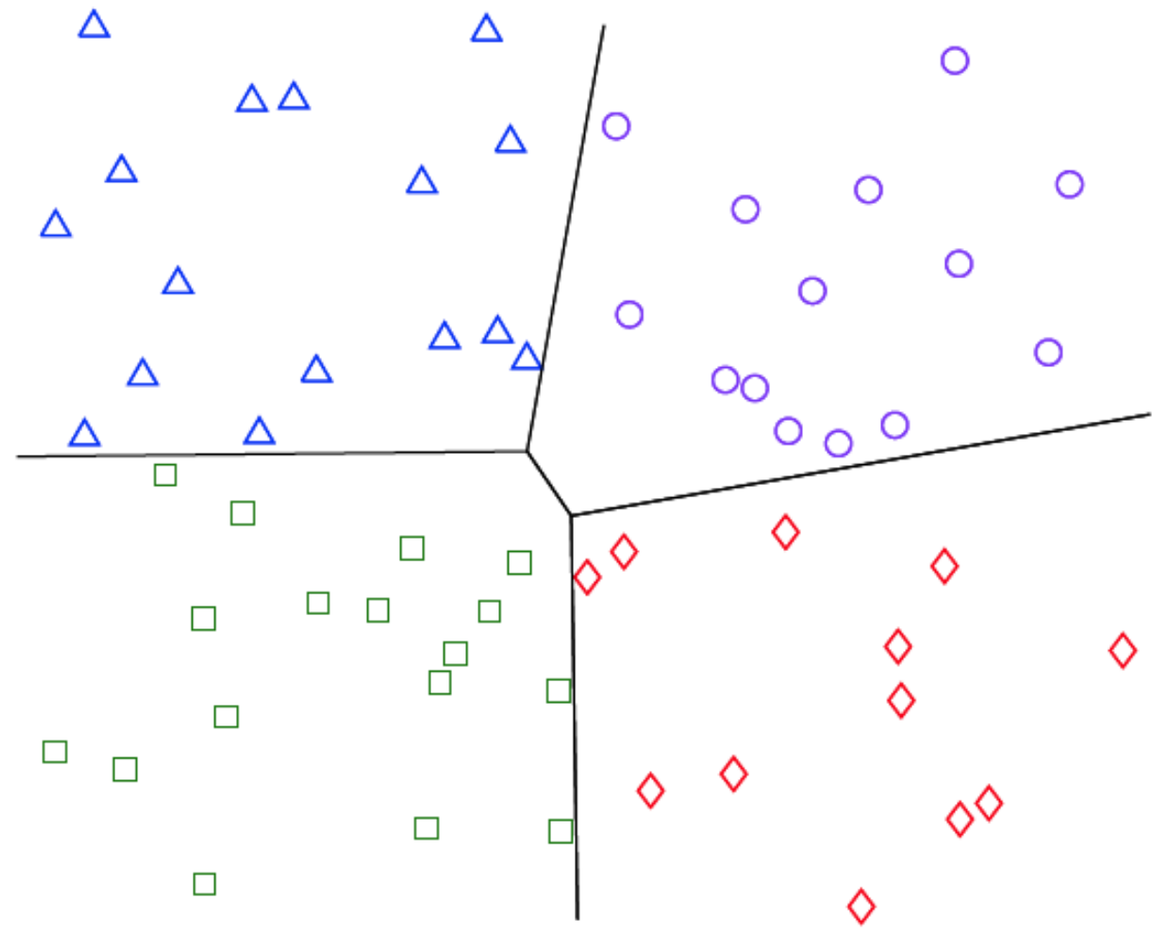
Data classification issues



Data classification issues



pairwise linear separation



multi-class linear separation