Lecture 6

Review

Normal game form

$P = \{1, \ldots, n\}$: set of players

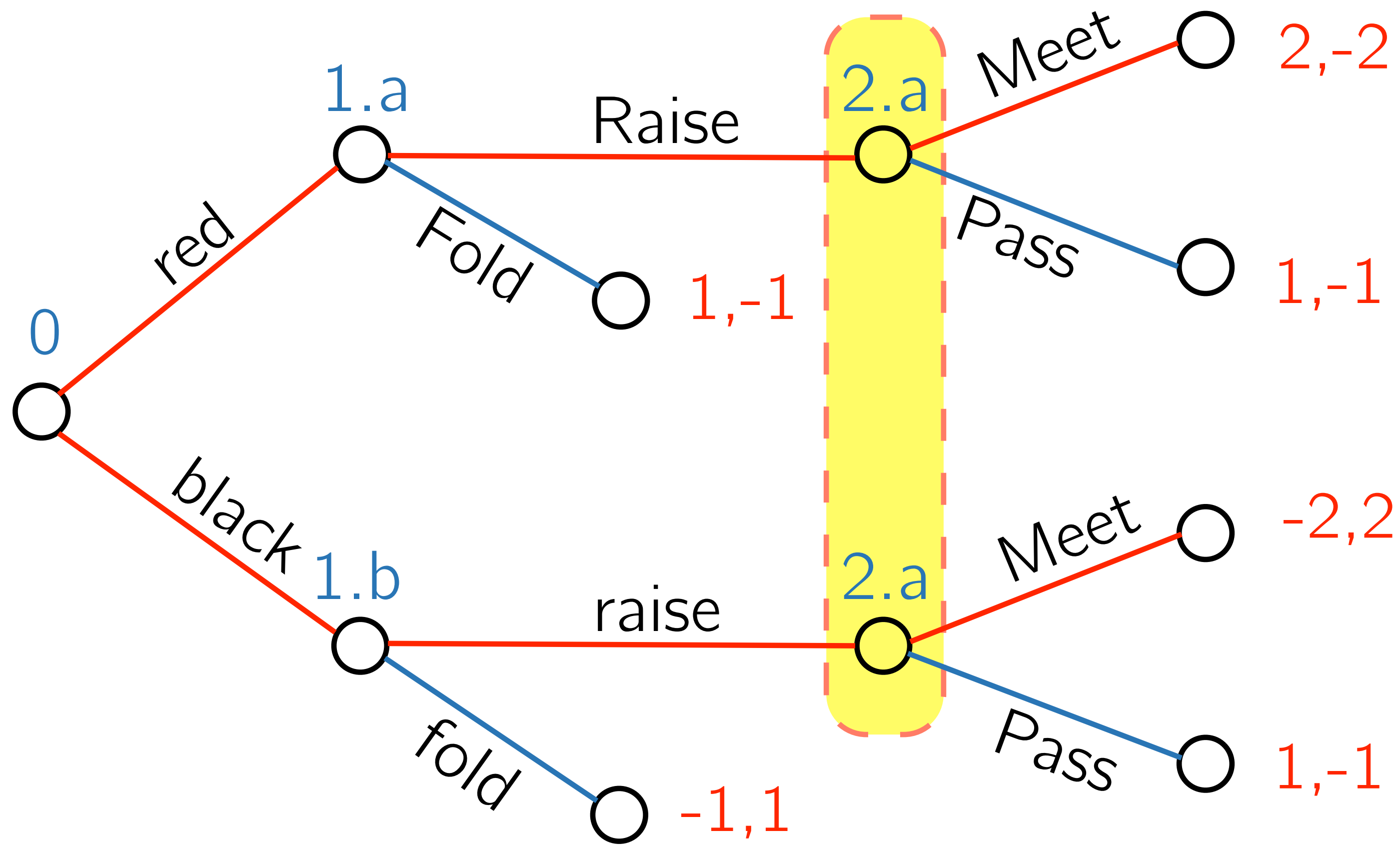
S_i : possible strategies for player i

$s = \{s_1, \ldots, s_n\}$: (pure) strategy profile of the game

S : set of strategy profiles

$\pi_i : S \mapsto \mathbb{R}$: payoff to player i when profile s is chosen by all players

Example



		P2	
		M	P
P1	Rr	0,0	1,-1
	Rf	0.5,-0.5	0,0
	Fr	-0.5,0.5	1,-1
	Ff	0,0	0,0

$S_1 = \{Rr, Rf, Fr, Ff\}$

$S_2 = \{M, P\}$

$s = \{Rr, M\}$

$\pi_1(s) = 0$

$\pi_2(s) = 0$

		P2	
		s_{12}	s_{22}
P1	s_{11}	0,0	1,-1
	s_{21}	0.5,-0.5	0,0
	s_{31}	-0.5,0.5	1,-1
	s_{41}	0,0	0,0

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$s_{1i}, \ldots, s_{ki}$: pure strategies for player i

$\sigma_i = p_{1i}s_{1i} + \ldots + p_{ki}s_{ki}$: mixed strategy for player i

p_{ji} : weight of strategy s_{ji} on σ_i

$\sigma = (\sigma_1, \ldots, \sigma_n)$: mixed strategy profile

Payoffs of mixed strategies

$$s_{1i}, \ldots, s_{ki} \; : \; \text{pure strategies for player } i$$

$$\sigma_i = p_{1i}s_{1i} + \ldots + p_{ki}s_{ki} \; : \; \text{mixed strategy for player } i$$

$$p_{ji} \; : \; \text{weight of strategy } s_{ji} \text{ on } \sigma_i$$

$$\sigma = (\sigma_1, \ldots, \sigma_n) \; : \; \text{mixed strategy profile}$$

Payoff for player 1

$$\pi_1(\sigma) = \sum_{s_{j1} \in S_1} \sum_{s_{j2} \in S_2} \ldots \sum_{s_{j2} \in S_2} p_{j1} p_{j2} \ldots p_{jn} \pi_1(s_{j1}, s_{j2}, \ldots, s_{jn})$$

Today

- The Fundamental Theorem of game theory
- How to find Nash equilibria
- Pareto and social optimality
- Weakly donated v. strictly dominated Strategies
- Dynamic games

The Penalty-Kick Games

		Goalie: left	Goalie: right
Kicker	left	0.58, -0.58	0.95, -0.95
	right	0.93, -0.93	0.70, -0.70

- What makes the kicker indifferent between his two options?
- That the goalie moves left with probability 0.39
- What makes the goalie indifferent between his two options?
- That the kicker moves left with probability 0.42

Empirical study: goalies dive left a 0.42 fraction of the time

Other notions of optimality

		Your partner	
		presentation	exam
You	presentation	90, 90	86, 92
	exam	92, 86	88, 88

- A strategy profile is Pareto-optimal if there is no other choice of strategies in which all players receive payoffs at least as high, and at least one player receives a strictly higher payoff

- A strategy profile is a social welfare maximizer (or socially optimal) if it maximizes the sum of the players' payoffs.

Next class

- Evolutionary game theory
- Fitness as a result of interaction
- Evolutionarily stable strategies (ESS)

Then

- Modeling network traffic using games