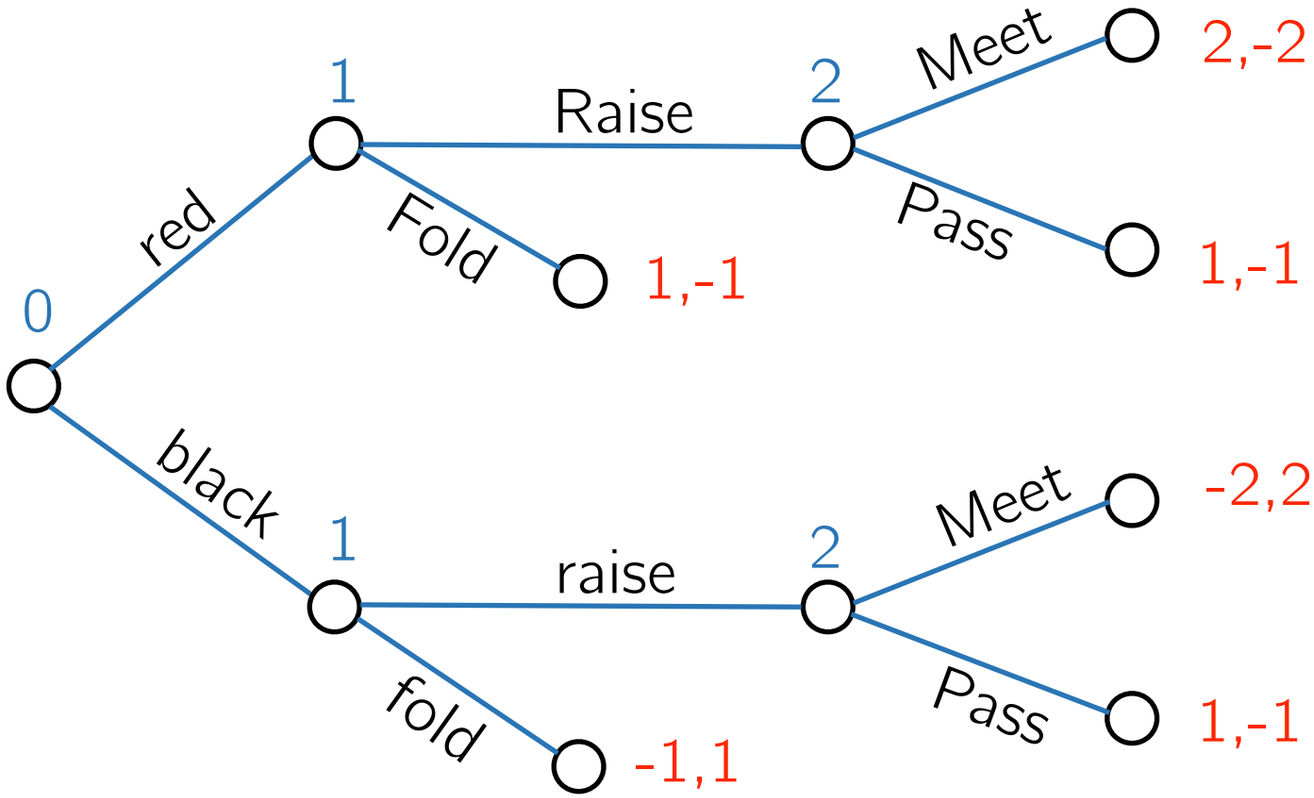
Lecture 5

Review

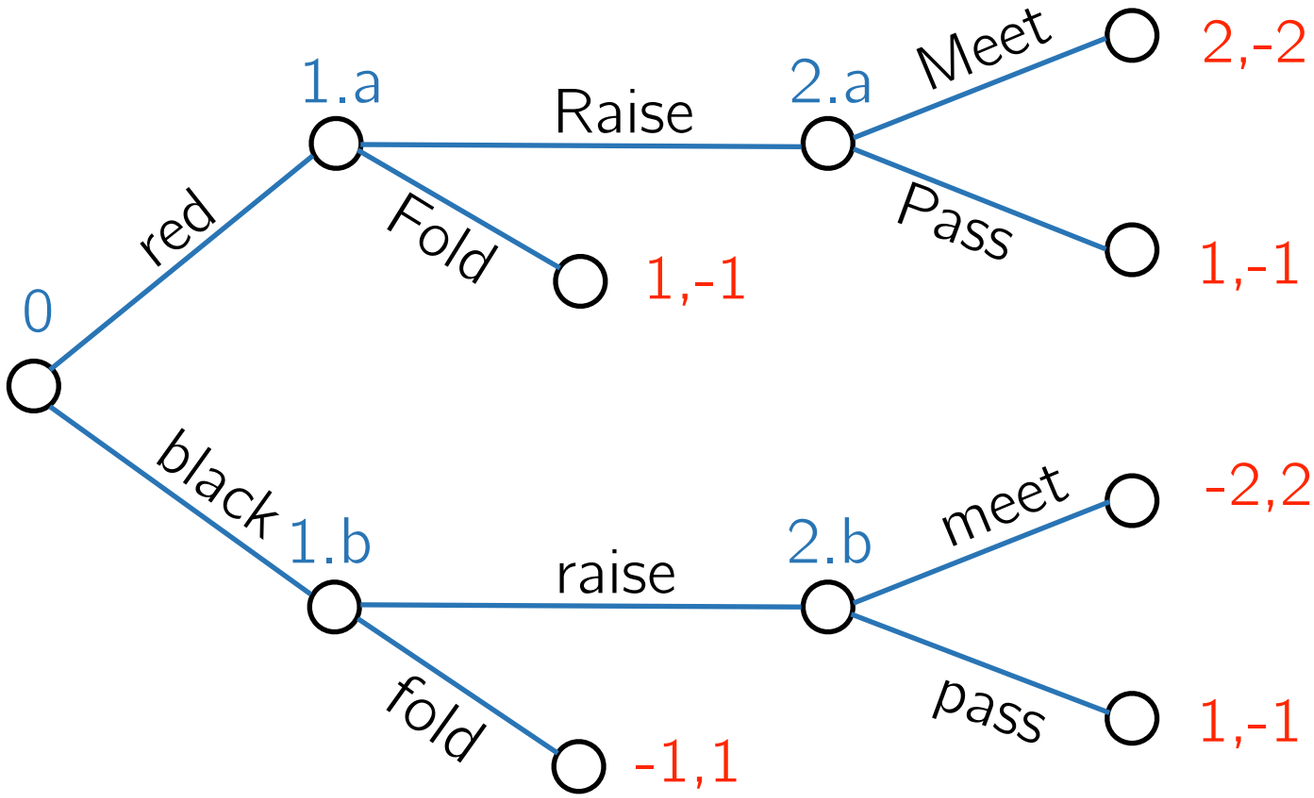
Card game



- P_1 takes the money if the card is red
- P_2 takes the money if the card is black

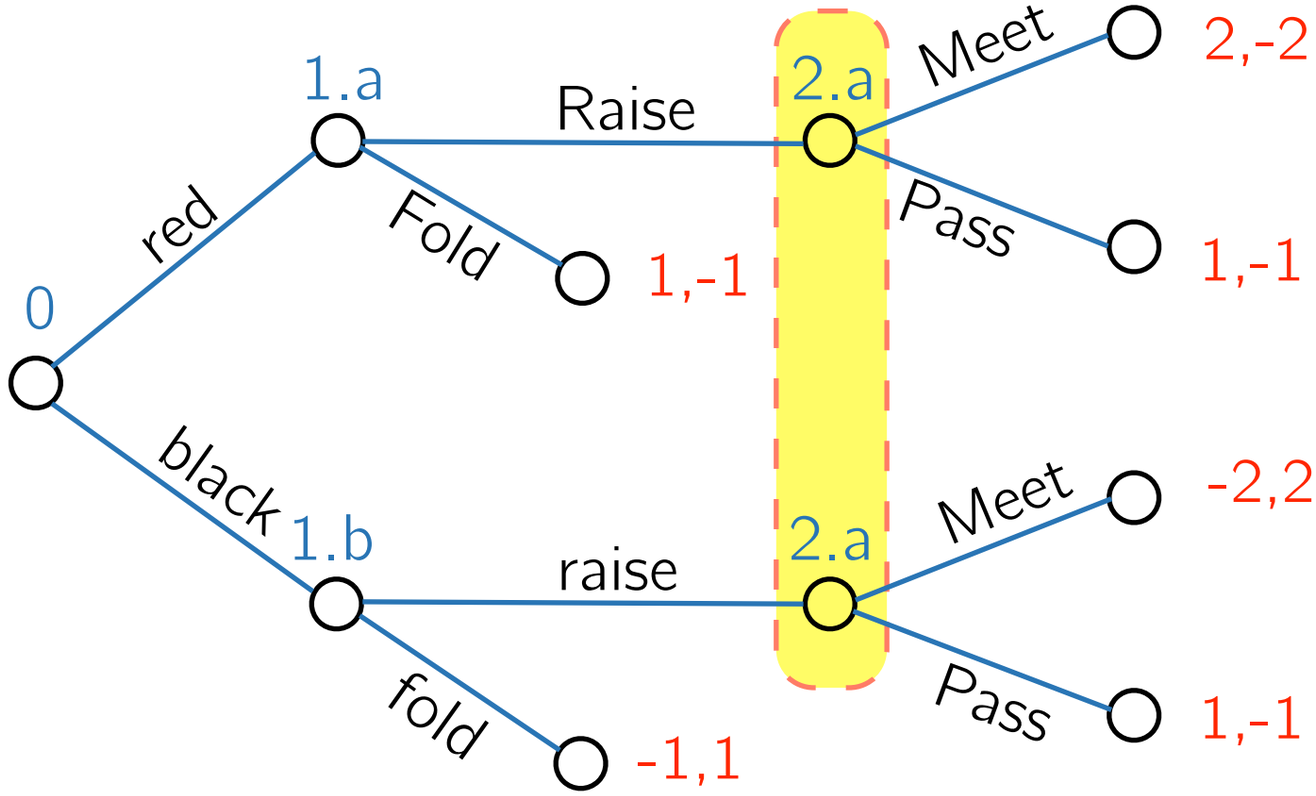
What is missing? Information state...

Perfect information



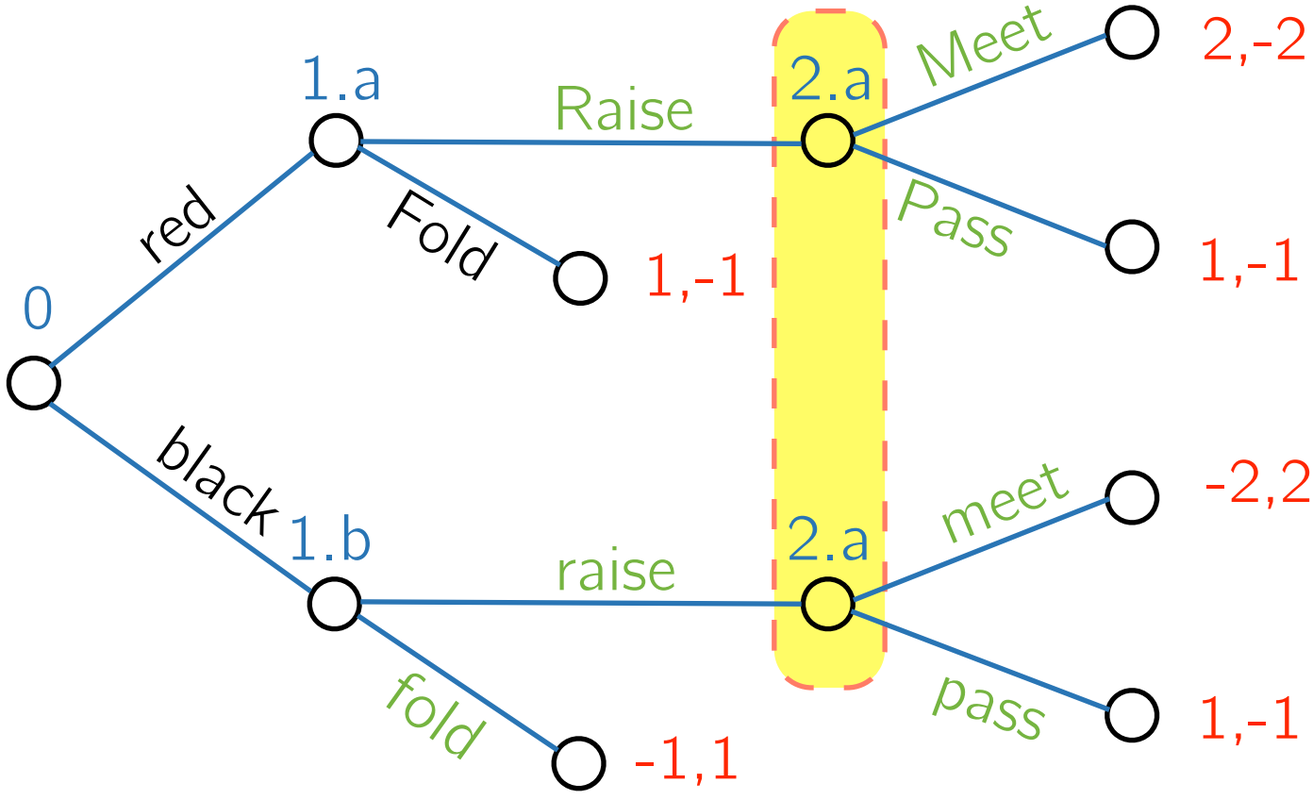
- P_1: 2 information states (4 choices - R, F, r, f)
- P_2: 2 information states (4 choices - M, P, m, p)

Imperfect information



- P_1: 2 information states (4 choices - R, F, r, f)
- P_2: 1 information state (2 choices - M, P)

Strategies v. actions

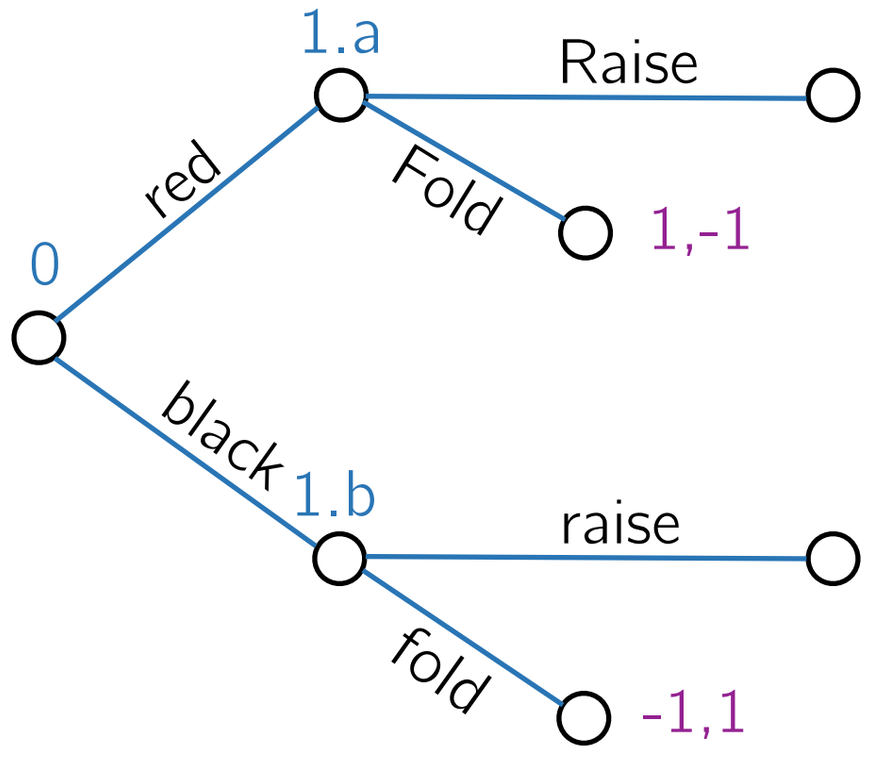


Action = move, choice, decision by players

≠ strategy

Strategy (P_1)

- Any rule for determining a move at every possible information state
- Maps information state into moves
- P_1 has 4 possible strategies: $s_1 \in \{Rr, Rf, Fr, Ff\}$

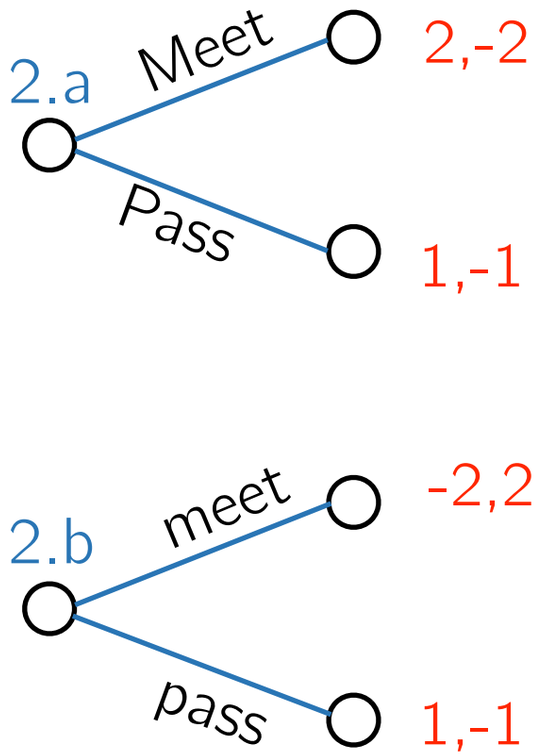


Strategy (P_2)

P_2 has 4 possible strategies

$s_2 \in \{Mm, Mp, Pp, Pm\}$

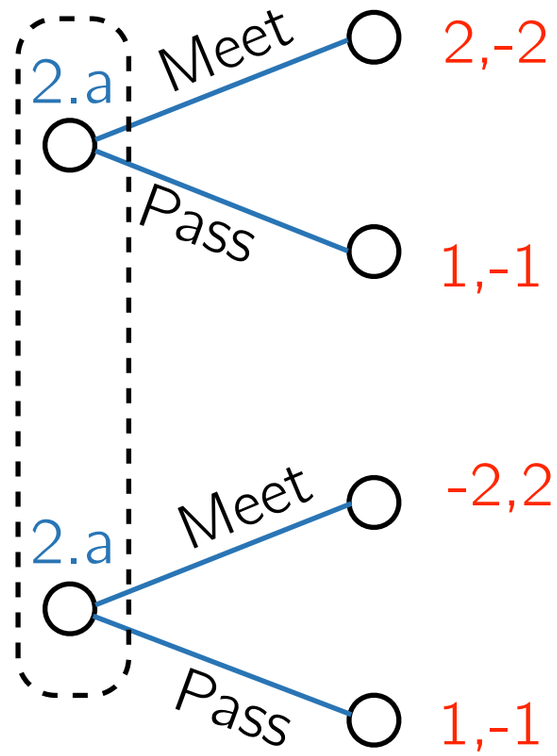
(Perfect information)



P_2 has 2 possible strategies

$s_2 \in \{M, P\}$

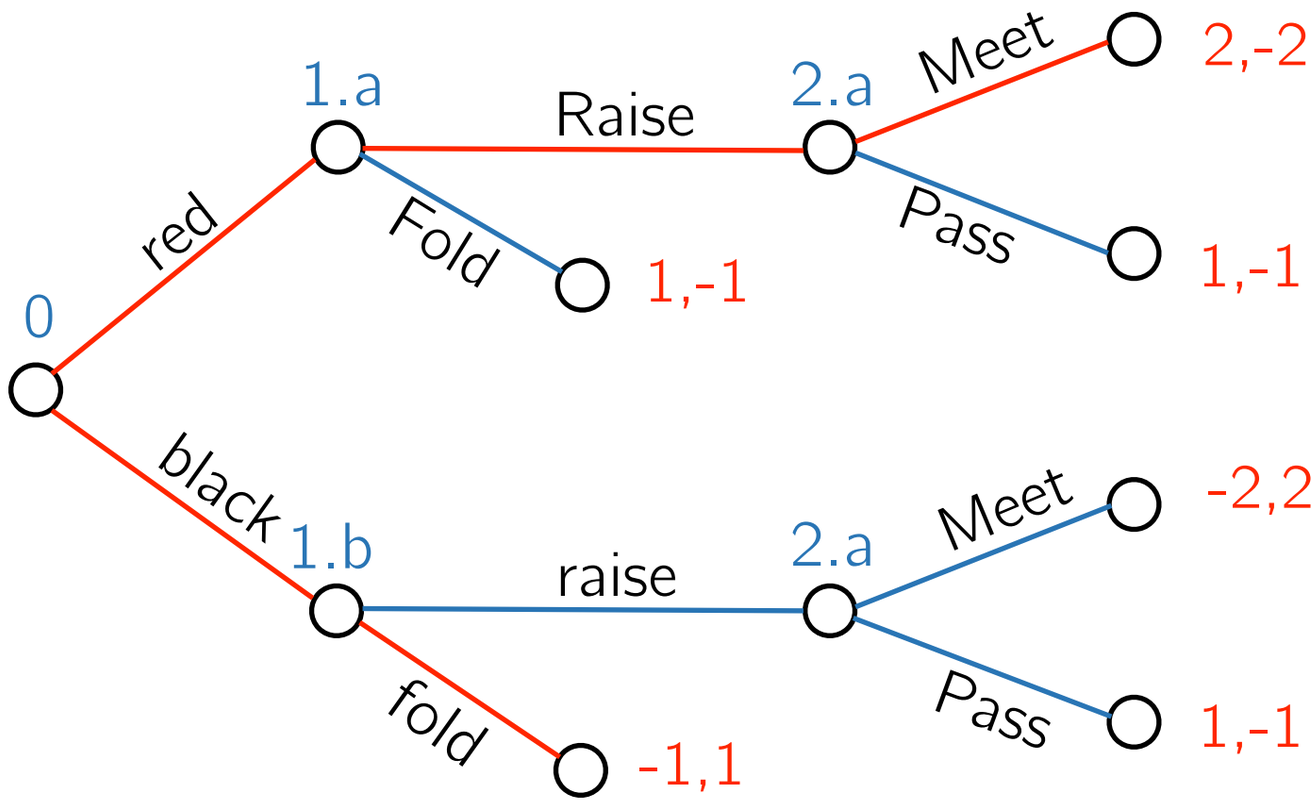
(Imperfect information)



If we know the players' strategies, can we predict the outcome?

P_1 plays Rf and and P_2 plays M

- P_2: 1 information state (2 choices)



Expected payoff?

Expected payoffs

- Strategy profile: $s = (s_1,\ldots,s_n)$
- Take, $s = (s_1, s_2) = (Rf,M)$ (cards have equal probability)

$$\pi_1(s) = 2\frac{1}{2} + (-1)\frac{1}{2} = \frac{1}{2}$$
$$\pi_2(s) = (-2)\frac{1}{2} + 1\frac{1}{2} = -\frac{1}{2}$$

- Expected payoffs to each player from each pair of strategies:

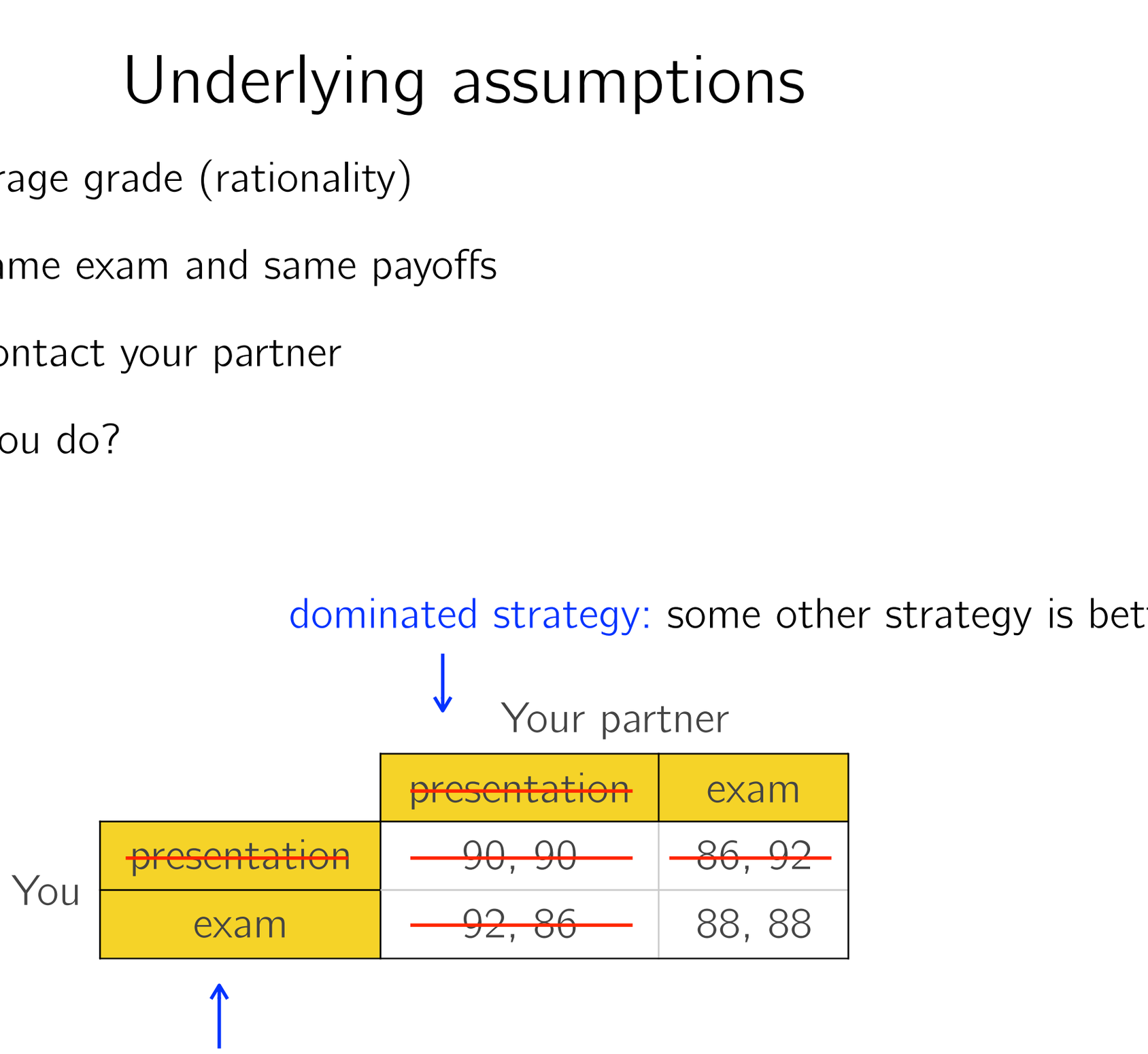
P_1 \ P_2	M	P
Rr	0,0	1,-1
Rf	0.5,-0.5	0,0
Fr	-0.5,0.5	1,-1
Ff	0,0	0,0

game in normal form!

Underlying assumptions

- Maximize average grade (rationality)
- Partner has same exam and same payoffs
- Not able to contact your partner
- What would you do?

dominated strategy: some other strategy is better



strictly dominant strategy: better than all other alternatives

Other coordination games

Balanced coordination

		Your partner	
		Power Points	Keynote
You	Power Point	1, 1	0, 0
	Keynote	0, 0	1, 1

Unbalanced coordination

		Your partner	
		Power Points	Keynote
You	Power Point	1, 1	0, 0
	Keynote	0, 0	2, 2

Battle of the sexes

		Your partner	
		Power Points	Keynote
You	Power Point	1, 2	0, 0
	Keynote	0, 0	2, 1

A Three-Client Game

- If the two firms approach the same client, then the client will give half its business to each
- Firm 1 is too small to attract business on its own, so if it approaches one client while Firm 2 approaches a different one, then Firm 1 gets a payoff of 0
- If Firm 2 approaches client B or C on its own, it will get their full business. However, A is a larger client, and will only do business with the firms if both approach A.
- Because A is a larger client, doing business with it is worth 8 (and hence 4 to each rm if it's split), while doing business with B or C is worth 2 (and hence 1 to each firm if it's split).

Nash: best response given what the other firm does

No strictly dominant strategy

No straggly dominated strategy

		Firm 2		
		A	B	C
Firm 1	A	4, 4	0, 2	0, 2
	B	0, 0	1, 1	0, 2
	C	0, 0	0, 2	1, 1

Back to presentation-exam...

- Maximize average grade (rationality)
- Partner has same exam and same payoffs
- Not able to contact your partner
- What would you do?
- Nash: Best strategy, given your partner's decision

		Your partner	
		~~presentation~~	exam
You	~~presentation~~	~~90, 90~~	~~86, 92~~
	exam	~~92, 86~~	88, 88

Nash is not necessarily the best outcome!

Today

- Formalize Mixed Strategies
- Formalize Nash Equilibrium
- The Fundamental Theorem of game theory
- How to find find Nash equilibria

Next class

- Pareto and social optimality
- Weakly donated v. strictly dominated Strategies
- Dynamic games

Then:

- Evolutionary game theory
- Fitness as a result of interaction
- Evolutionarily stable strategies

Then:

- Modeling network traffic using games