# Lecture 10

# Review

**Descending-bid auctions**

process: bidders learn nothing while auction is running

payoff: winner pays first price $b_i$ at which bidder breaks silence

**Ascending-bid auctions**

process: bidders do not stay in auction after price exceeds true value; do not drop out before the price reaches the true value

payoff: bidder's "drop-out price" equals her $b_i$

**First-price sealed-bid auctions**

**Second-price sealed-bid auctions**

First-price sealed-bid auctions bidders will tend to bid lower. Why?

# Second-price auctions

- Auction form on eBay
- Keyword-based advertising in search engines (Google, Yahoo!)
- Bidders = players
- Player $i$ values the item $v_i$ *and bids* $b_i$
- Strategy profile: $s = (b_1, \ldots, b_n)$
- Payoff to a player:

$$\pi_i(s) = \begin{cases} 0, & \text{if player } i \text{ is not the winning bid} \\ v_i - b_j, & \text{otherwise} \end{cases}$$

where $b_j$ is the second-best bid

# Remarks

- Bid determines whether you win or lose, but not how much you pay
- "Truthfulness" is a dominant strategy
- Makes sense to bid true value, even if other overbid, underbid, collude, etc.
- Not the case for other types of auctions
  - First-price sealed-enveloped auctions

$$\pi_i(s) = \begin{cases} 0, & \text{if player } i \text{ is not the winning bid} \\ v_i - b_i, & \text{otherwise} \end{cases}$$

$$\pi_i(s) = 0 \text{ if you pay what the item is worth to you}$$

  - All-pay auctions

$$\pi_i(s) = \begin{cases} -b_i, & \text{if player } i \text{ is not the winning bid} \\ v_i - b_i, & \text{otherwise} \end{cases}$$

In general you want to bid below your true value

# First-price sealed-enveloped auctions

- Number of bidders is known
- Partial information on values, but not the values exactly
- Consider two bidders and values $v_i$ distributed on $[0, 1]$ (common knowledge)
- A function $s(v) = b \geq 0$ maps true value to non-negative bid $b$

**Assumptions:**

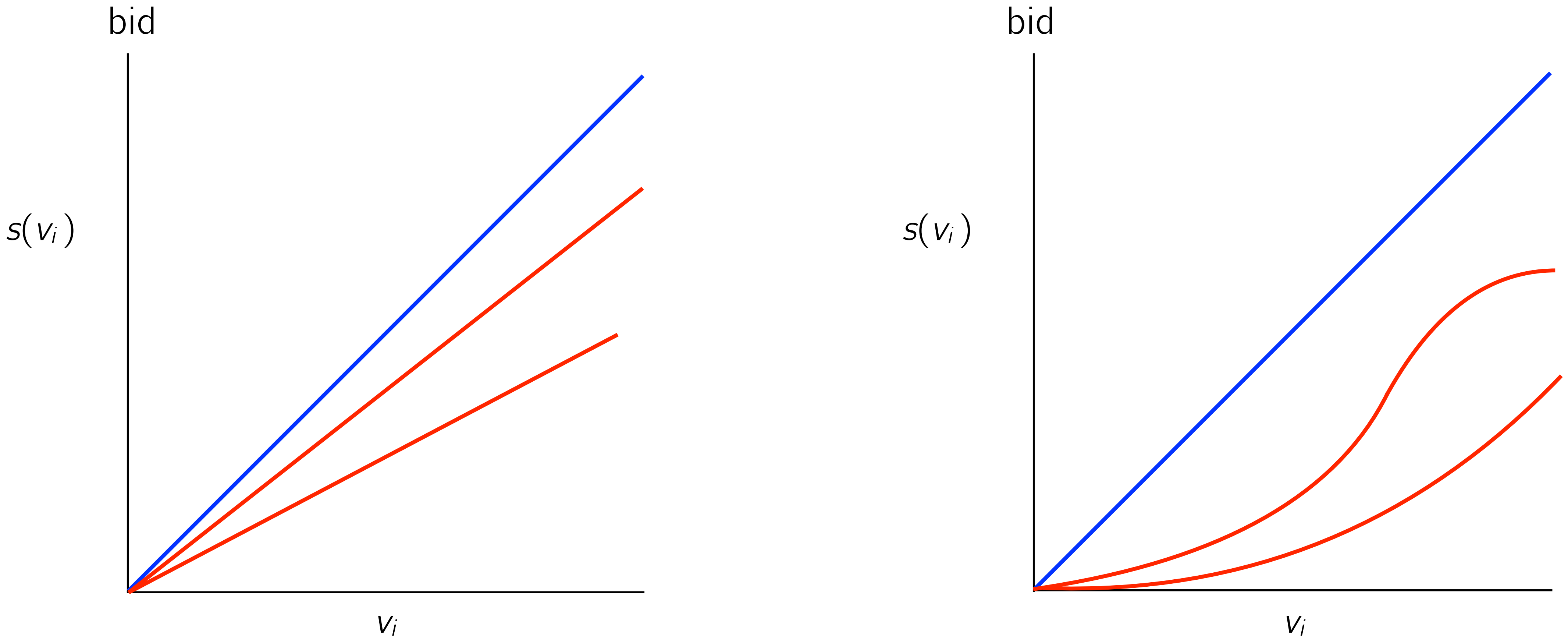
- Both players follow the same strategy $s(\cdot)$
- $s(\cdot)$ is strictly increasing and differentiable
  $\rightarrow$ different true values lead to different bids
- $s(\cdot) \leq v$ for all $v$ and $s(0) = 0$

narrow the search for equilibrium strategies

- Highest value produces highest bid, but never bid above true value

# Examples

- $s(v) = v$ : bid true value
- $s(v) = cv,\ c < 1$ : shade bid



# First-price sealed-enveloped auctions

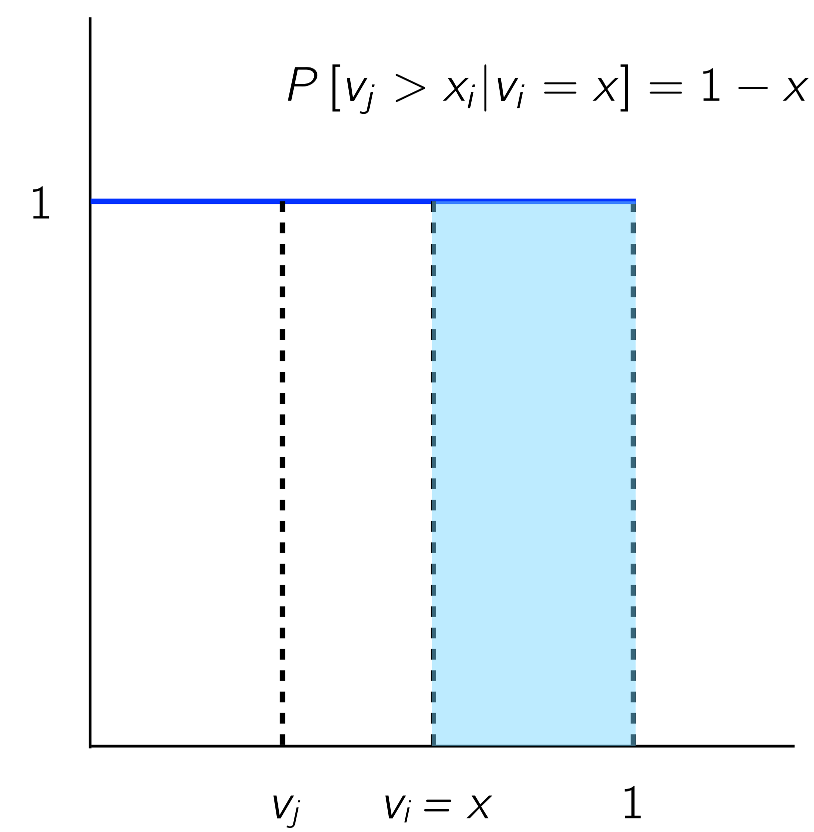
- Bidders are identical except for the true value they draw from the distribution
- Bidder $i$ will win bid with probability $v_i$
- If bidder i wins it receives payoff $v_i$ - $b_i$ = $v_i$ - $s(v_i)$
- Payoff to a player:

$$\pi_i(s) = \begin{cases} 0, & \text{if player } i \text{ is not the winning bid} \\ v_i(v_i - s(v_i)), & \text{otherwise} \end{cases}$$

probability of winning

true value

bid



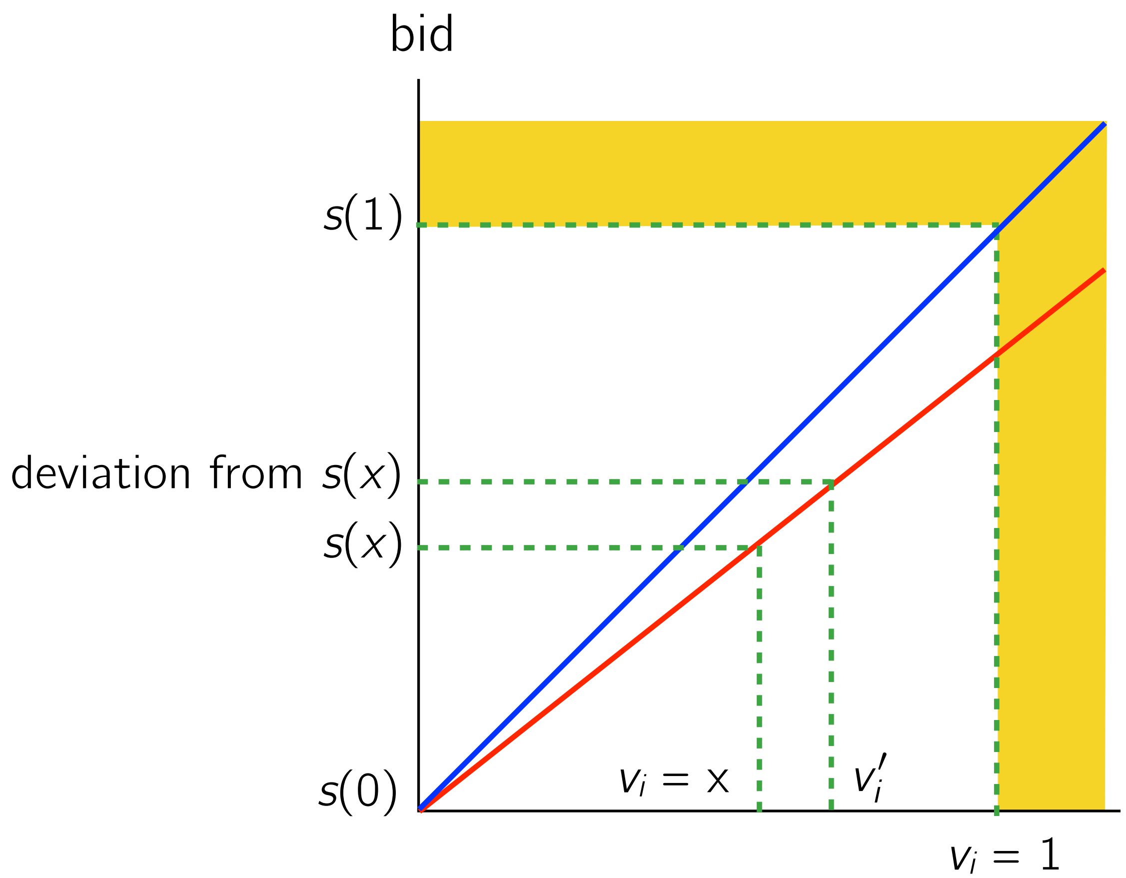

**Question:**

When is a strategy profile s = ($s(v_1)$,...,$s(v_n)$) a Nash?

# Revelation principle

$$\pi_i(s) = \begin{cases} 0, & \text{if player } i \text{ is not the winning bid} \\ v_i(v_i - s(v_i)), & \text{otherwise} \end{cases}$$

probability of winning    true value    bid



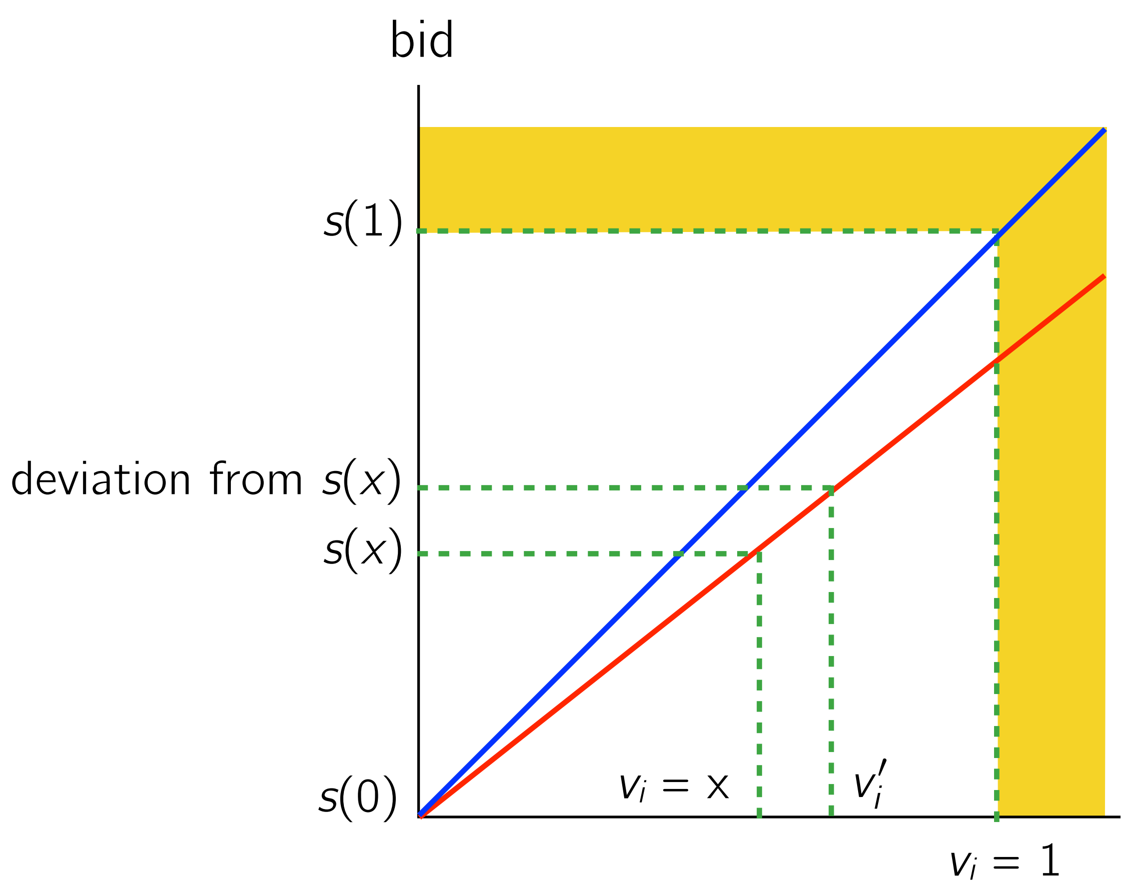

Player $i$ should never announce a bid above $s(1)$ since she wins with $s(1)$
+ gets a higher payoff

Deviations in bid can be interpreted as deviation by supplying different "true value"

# Revelation principle

$$\pi_i(s) = \begin{cases} 0, & \text{if player } i \text{ is not the winning bid} \\ v_i(v_i - s(v_i)), & \text{otherwise} \end{cases}$$

probability of winning    true value    bid




Player $i$ should never announce a bid above $s(1)$ since she wins with $s(1)$
+ gets a higher payoff

**Condition of best response:**

$$v_i(v_i - s(v_i)) \geq v_i'(v_i - s(v_i')) \text{ for all alternative "true values" } v_i'$$

# Nash strategy profile

- Consider $s(v) = v/2$
- Payoff if player $i$ is the winning bid (for true value $v_i$):

$$v_i \left( v_i - \frac{v_i}{2} \right) = \frac{v_i^2}{2}$$

- Payoff if player $i$ is the winning bid (for true value $v_i'$):

$$v_i' \left( v_i - \frac{v_i'}{2} \right) = v_i' v_i - \frac{(v_i')^2}{2}$$

- So that:

$$\frac{v_i^2}{2} - \left( v_i' v_i - \frac{(v_i')^2}{2} \right) = \frac{1}{2} \left( v_i - v_i' \right)^2 \geq 0$$

Bidding half your true value is a best response (if the other bidder is doing the same)

# Deriving the two-bidder equilibrium

**Let**

$$\pi_i(v) = v(v_i - s(v))$$

**Back to original condition**

$$v_i(v_i - s(v_i)) \geq v_i'(v_i - s(v_i')) \text{ for all alternative "true values" } v_i'$$

**Satisfying the condition requires that**

$$\left.\frac{d\pi_i(v)}{dv}\right|_{v=v_i} = \left.\left(v_i - s(v) + v\frac{ds(v)}{dv}\right)\right|_{v=v_i} = 0$$

$$\left.\frac{ds(v)}{dv}\right|_{v=v_i} = 1 - \frac{s(v_i)}{v_i} \text{ for all } v_i \in [0,1]$$

**which is solved by the function**

$$s(v_i) = \frac{v_i}{2}$$

# Today

- Little experiment (4 participants)
- Power in social networks
- Bargaining Nash equilibrium
- The ultimatum game

**Next class**

- Modeling network exchange
- Web search applications

# Key questions

- Is power a property of a individual?
- Is a node particularly powerful given its relative position in a social network?
- Is power a property of the relation between two nodes?
- Under what conditions does an individual have powerful over another?

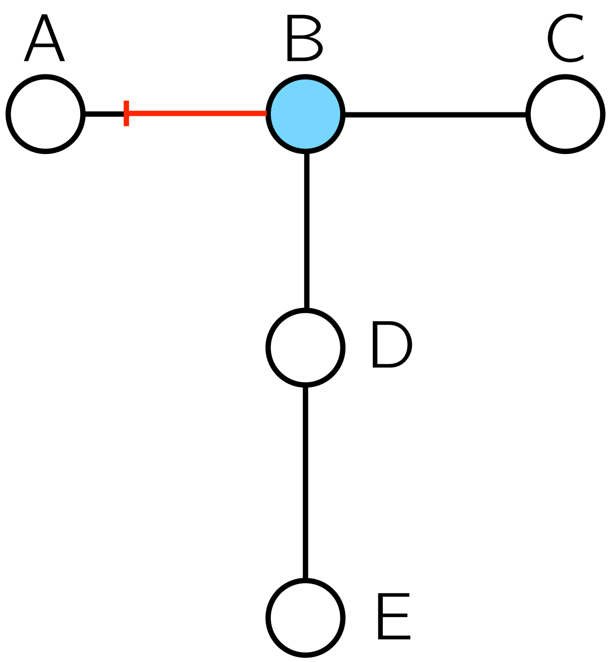
# The notion of power

- Nodes = individuals
- Links = relationships between individuals
- Relationships = social exchanges
- Example: favors, revenue of working together, psychological value of being friends, etc.
- A relationships produces value for the two individuals
- Power: imbalance in how value is divided
- Powerful party gets the majority of the value
- Scenario 1: imbalance is the result of the personalities involved
- Scenario 2: imbalance is a función of the larger network
  (rooted in network structure)

# Example

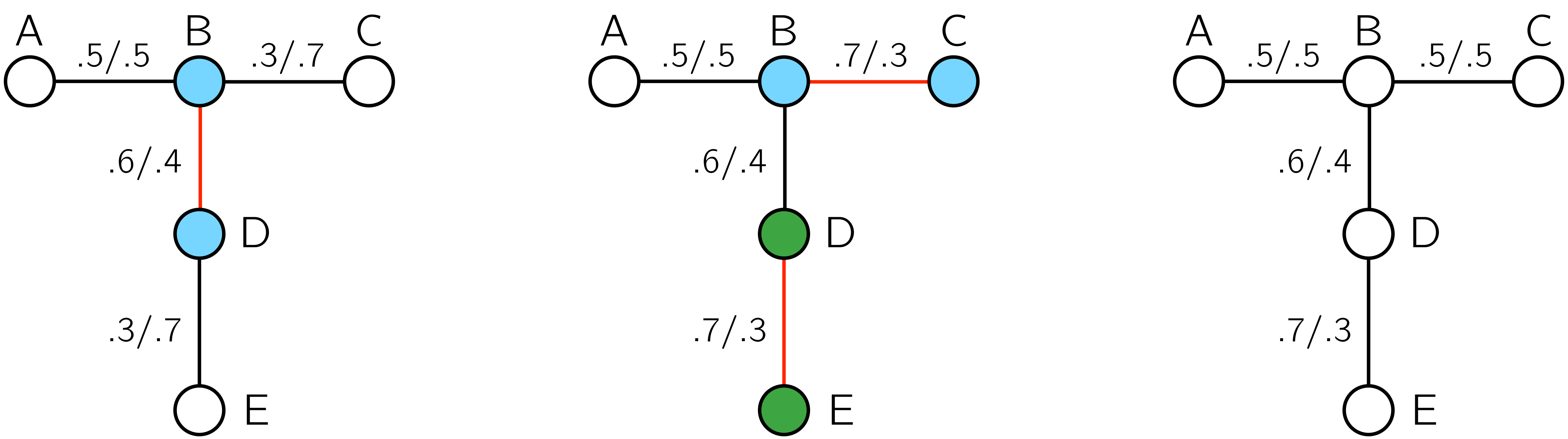
Node B holds powerful position based on:

- dependence as a source of value
  - nodes A and C depend on B
- exclusion (choose a “best friend”)
  - ability to form partnerships
  - node B has unilateral power
  - choose one of A and C
- betweenness (value flows though paths)
  - node B lies on paths lies all shortest paths (except D-E)
  - between access point between multiple different pairs
- others: clustering, local bridges, inter-community, etc.



# Experimental studies

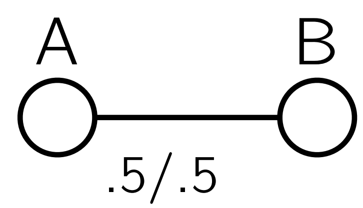
- Social network: money placed on each edge (value of a relationship = $1)
- Nodes joined by an edge negotiate how to share the money
- Each node takes part in the division with only one neighbor
- 1-exchange rule: choose "best friend" (at most one successful partnership)
- Node's decision: how much to share? + with whom?
- Multiple rounds




one successful partnership　　multiple successful partnerships　　no successful partnership

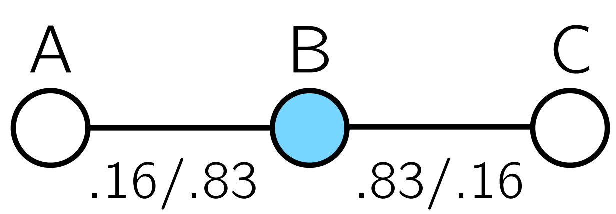
# Infer principles about power?

**2-node path**



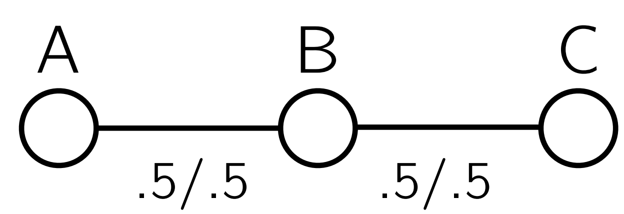

**3-node path**

1-exchange rule




B receives majority of the money

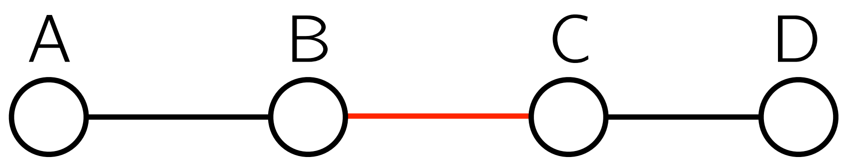
2-exchange rule

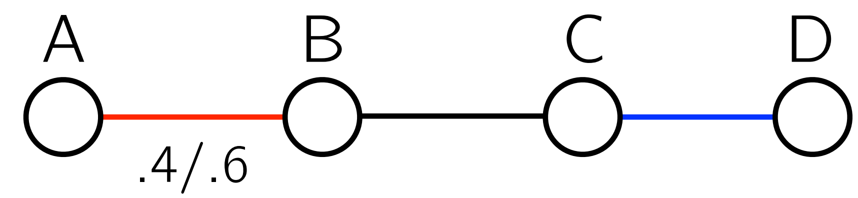



B needs A and C as much as they need B

# Infer principles about power?

**4-node path**

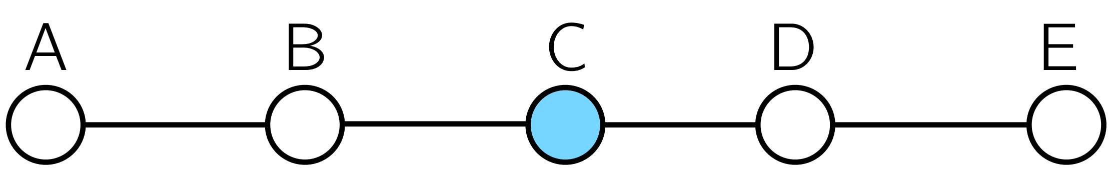



B's thread to exclude A is costly to execute

**5-node path**

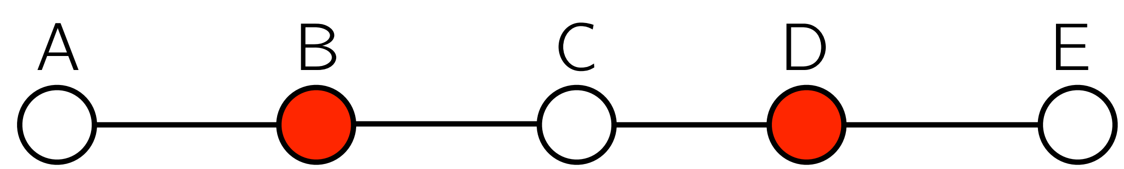
1-exchange rule




C has central position, but neighbors B and D have attractive alternatives (does slightly better than A or E)

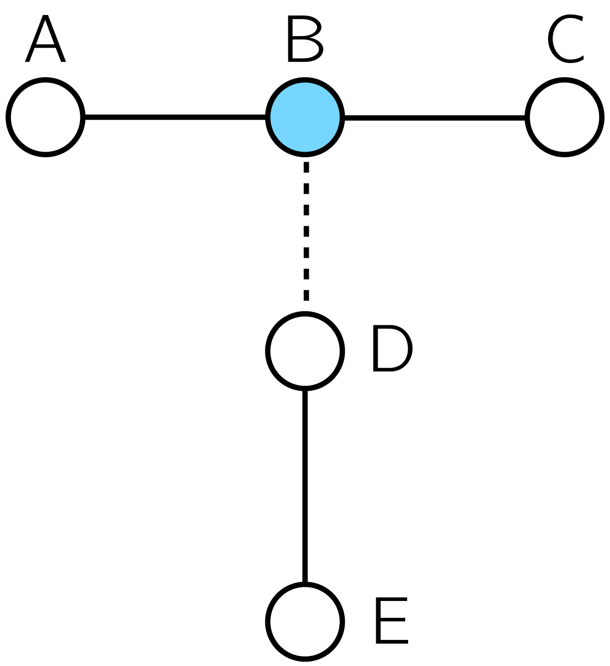
betweenness centrality can be misleading

2-exchange rule for B and D



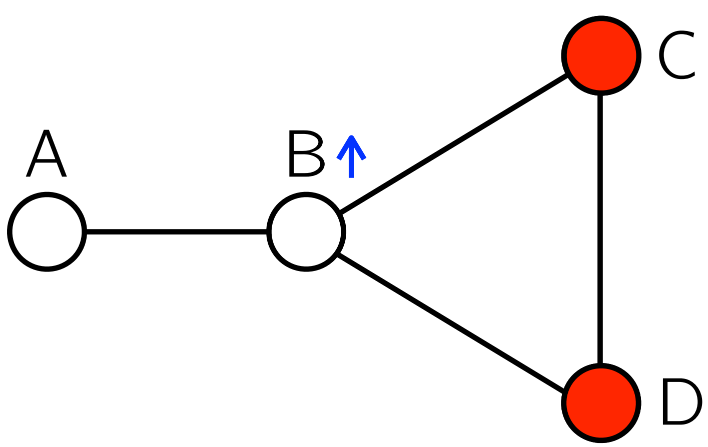

B and C need C to take part in two exchanges each

# Infer principles about power?



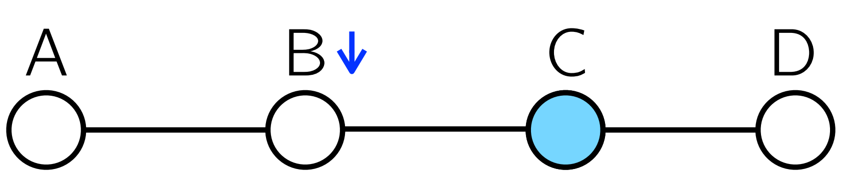

**T-graph**

- B can exclude A or C
- D does not have a realistic option besides E
- B and D almost never exchange
- B achieves high favorable exchanges

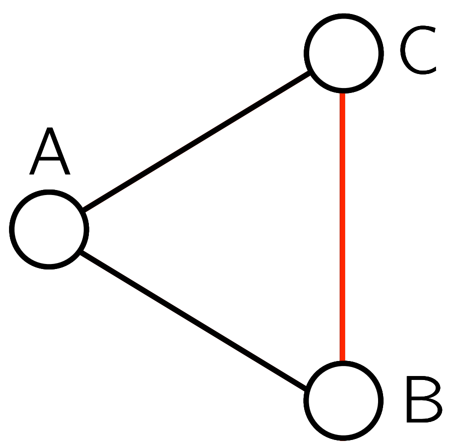



**Stem graph**

- B does better than in the 4-node path. Why?
- B does not have comparatively powerful neighbor

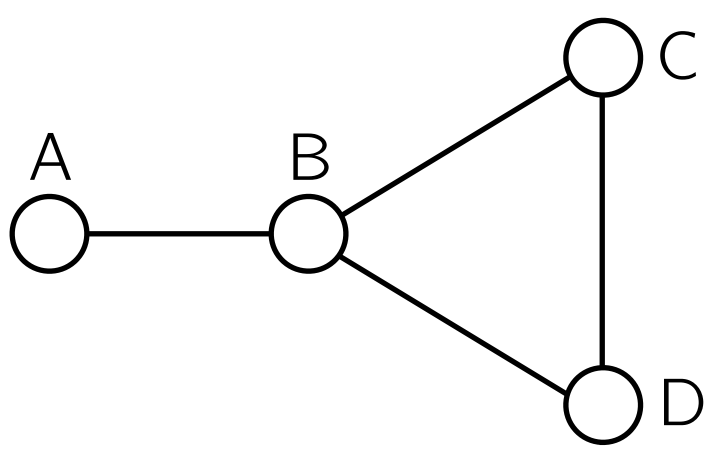


# Infer principles about power?

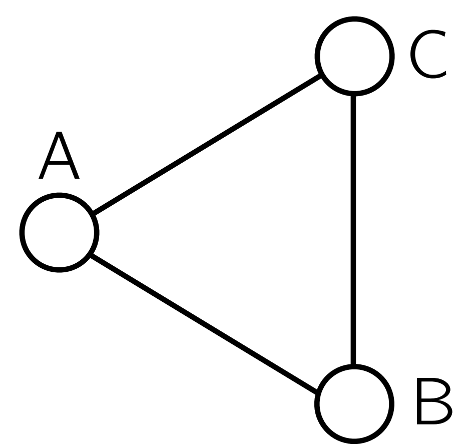



**Triangle**

- Indefinite cycle with some node always left out
- Nodes playing for "the last shoot"
- Outcome hard to predict




vs.




Node A allows for
predictable outcome

Node who is left out can do
something about it
(no stationary pattern emerges)

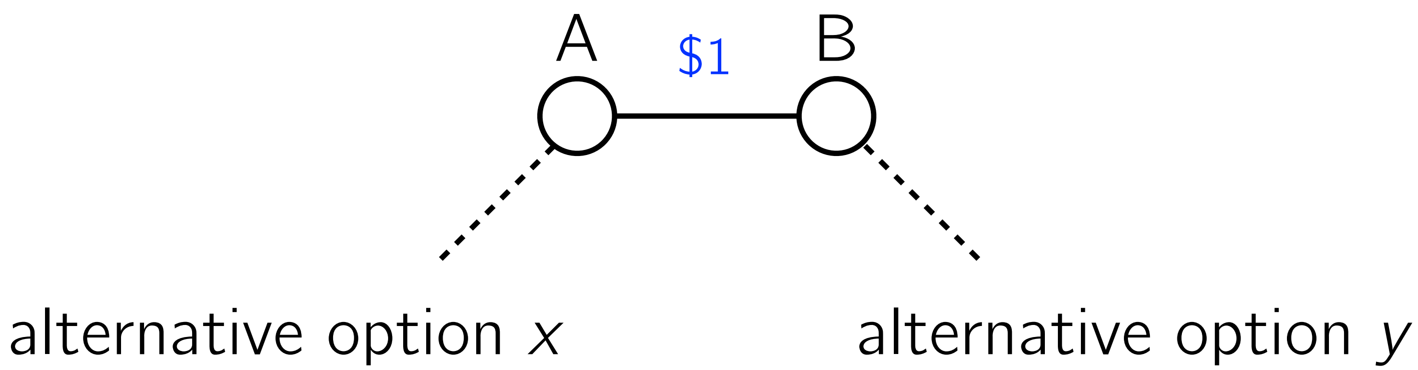
# 2-person interaction

- Exchanges take place on an arbitrary network
- Framework to express predictions
- Explain differences between:
  - equal and asymmetric divisions of value across an edge
  - strong and weak power (when do imbalances go to the extreme?)
  - stationary and non-stationary interaction patterns

**Two principles:**

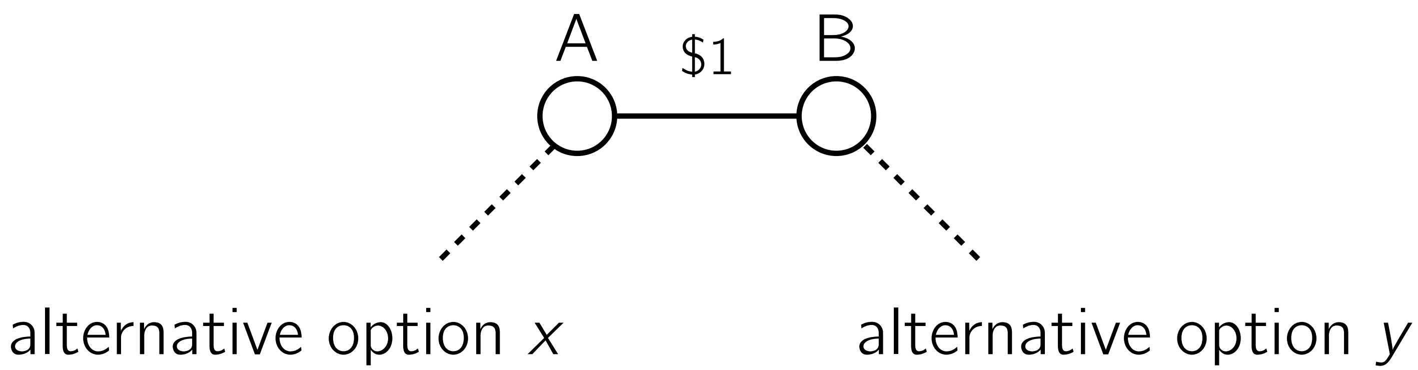
- The Nash bargaining solution (theoretical perspective)
- The ultimatum game (empirical perspective)

# The Nash bargaining solution




- Each node has an alternative or outside option
- If A (B) does not like deal with B (A) she can choose option $x$ ($y$ respectively)
- Deal-breaker for A:
  - She is going to get less than $x$ for negotiation with B
  - Node A wants to get at least $x$
- If $x + y > 1$ then no deal is possible. Why?
- Cannot divide \$1 so that one gets at least $x$ and the other at least $y$
- Assumption: $x + y \leq 1$

# The Nash bargaining solution



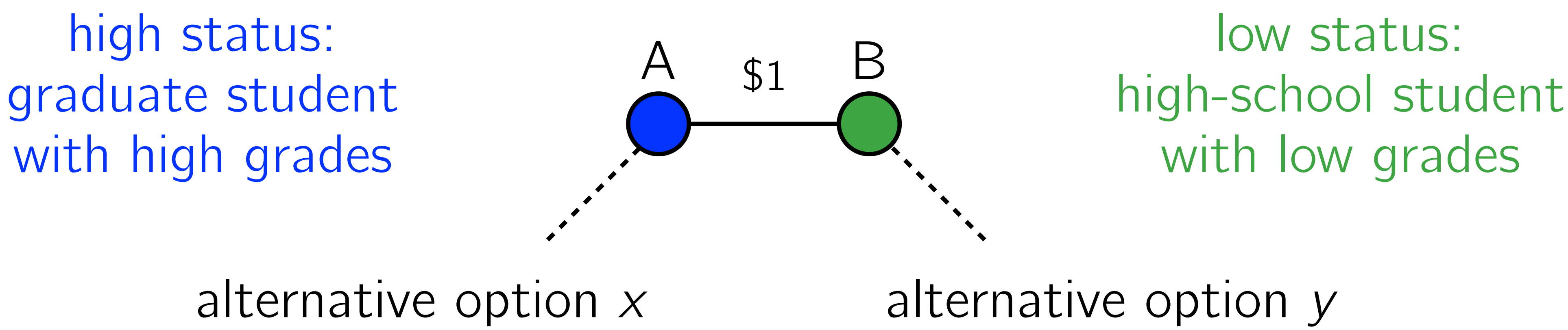

- How to split surplus $s$ = 1 - $x$ - $y$?
- Note that $s \geq 0$ since $x + y \leq 1$
- If A and B have equal bargaining power then split surplus evenly
- Payoff to each player:

$$\pi_A = x + \frac{1}{2}s = \frac{x + 1 - y}{2}$$
$$\pi_B = y + \frac{1}{2}s = \frac{y + 1 - x}{2}$$

Each player receives minimum value

- Each node depends equally on the other to make negotiation work
- Equidependent outcome

# Remarks

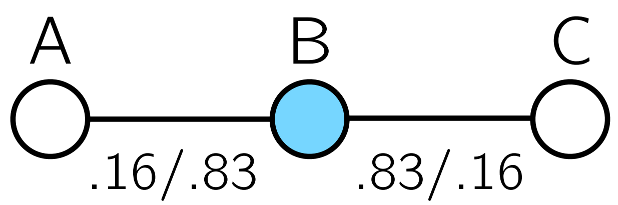



- Power arises though differences in outside options
- Ensure that you have as strong an outside option as possible
- Believe negotiating with higher and lower status partners?
- With lower-status partner: people tend to inflate size of their outside option $x$
- With higher-status partner: people tend to reduce size of their outside option $y$

Subject believe to be higher-status tends to achieve better bargaining outcomes than theoretical predicted

# The ultimatum game

1-exchange rule




- Nodes A and C do not have outside options
- B receives majority of the money ... but not all of it!
- Why not a completely unbalanced outcome?

- Value ($1) assigned to A who has to propose a division to B
- Node B has the option to approve or reject
- If B approves, they honor the devision
- If B rejects each node gets nothing
- One-time interaction
- How should they behave?
- Money-maximizing (rational) agents
- If A any any positive amount ($.99), B should accept ($.01>0)
- Not how humans behave

# The ultimatum game

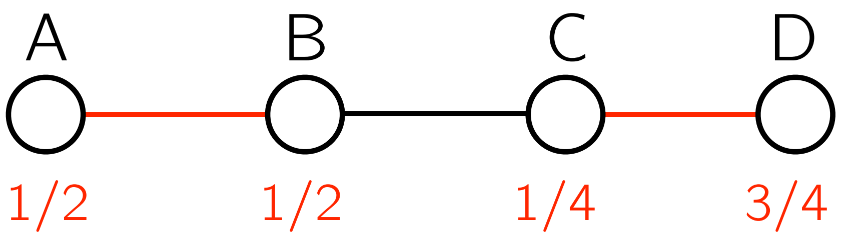
- Most people playing A offer 1/3
- A significant number playing A offer 1/2
- Very unbalanced offers are rejected
- Interesting cultural variations
  - levels of offer differ
  - probability of being rejected not
- Tendency toward balanced divisions remains
- Reconcile with game-theoretic framework?
  - payoff reflects complete evaluation of given outcome
  - care about treatment: 10% of total → significant negative emotional payoff
  - node A tends to offer relatively balanced to avoid reject
- People's payoffs are not well modeled by strict money-maximization

# Modeling network exchange

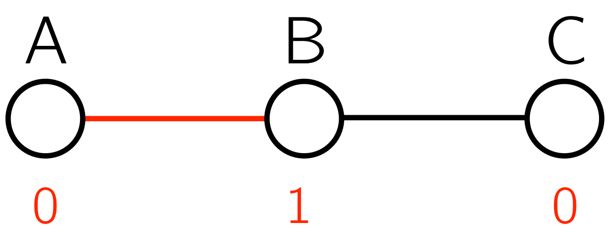
- Approximately predict outcome of network exchange
- Outcome:
  - matching set of nodes (who exchanges with whom?)
  - a value indicating how much a node gets from its exchange
- If two nodes are matched, the sum of their values is equal to 1
- If a node is not matched, then her value is 0 since there is no exchange
- Stable and not stable outcomes

# Stable and non-stable outcomes

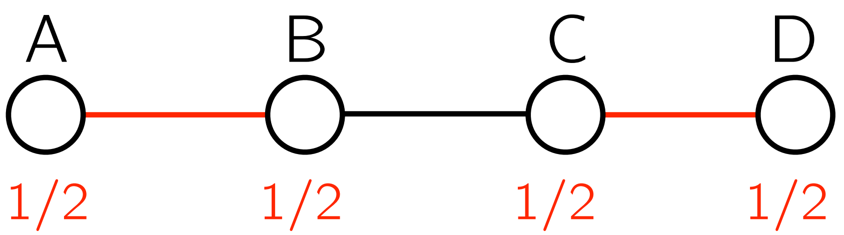
**Stable outcome:** no node X can propose an offer to some other node Y that makes both X and Y better off.



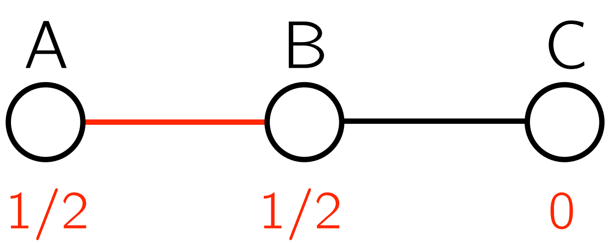

unstable




stable




stable

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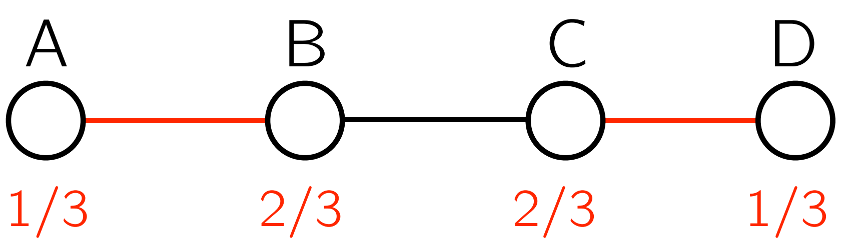
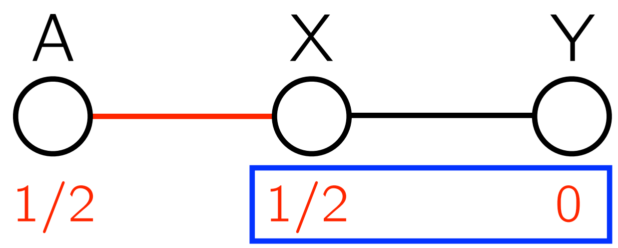



stable

# Instability of a network

**Instability:** Given an outcome consisting of a matching and values for the nodes, an instability in this outcome is an edge not in the matching joining two nodes X and Y, such that the sum of X's and Y's values is less than 1

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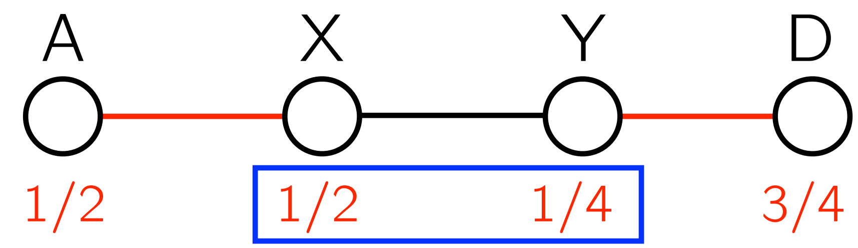


unstable

nodes X and Y have the opportunity + incentive to disrupt the status quo

# Next Class

- Modeling network exchange
- Web search applications