Lecture 13

- can I estimate the state? -

Review

- Define reachability of a system
- Give test for reachability of linear systems
- State feedback for linear systems

Analysis and design of systems

non-linear systems

Lyapunov stability (general theory)

Theorems (Lyapunov/Lasalle)

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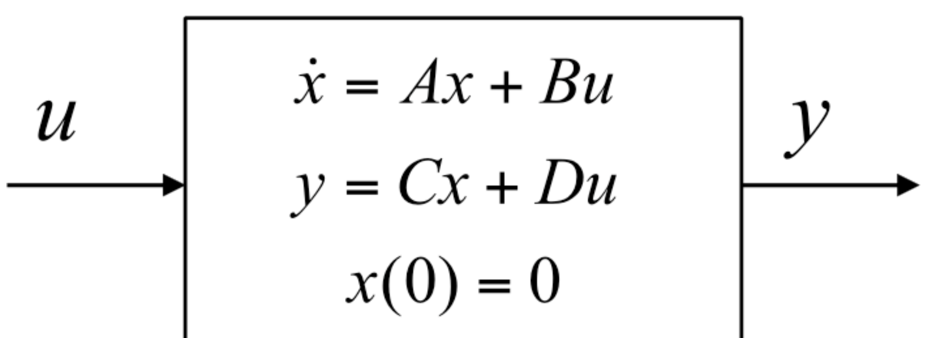
linear systems

Lyapunov stability (special case)

Theorems (eigenvalues)

local description of non-linear dynamics



Aleksandr Mikhailovich Lyapunov

Born	June 6, 1857 Yaroslavl, Imperial Russia
Died	November 3, 1918 (aged 61)
Residence	Russia
Nationality	Russian
Fields	Applied mathematics
Institutions	Saint Petersburg State University Russian Academy of Sciences Kharkov University
Alma mater	Saint Petersburg State University
Known for	Lyapunov function

“The general problem of the stability of motion”

-> start of stability theory

Properties of linear systems

- Stability characterized by eigenvalues
- Many applications and design tools available

Design concepts

- Stabilization: stabilize the system around an equilibrium point

  Given an equilibrium point, find a "control law" $u = \alpha(x)$

  such that $\lim_{t \to \infty} x(t) = x_e$ for all $x(0) \in \mathbb{R}^n$.

Control law will depend on the state!

Goal: Find a liner control law

$$u = -Kx$$

such that the closed-loop system

$$\dot{x} = Ax + Bu = (A - BK)u$$

is stable at $x_e = 0$.

- caveat -

you need to know if a system is reachable before you try to stabilize it

Reachability test

Reachability test (for linear system)
Consider a linear system of the form

$$\frac{dx}{dt} = Ax + Bu$$

The system is reachable if and only if the reachability matrix

$$W_r = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

is invertible. That is, if and only if W_r is of full rank.

Theorem
Consider a linear system of the form

$$\frac{dx}{dt} = Ax + Bu$$

with input $u = -Kx$. The eigenvalues of $(A - BK)$ can be set to arbitrary values if and only if the pair (A, B) is reachable.

Today

- Define observability
- Give conditions for linear systems
- Introduce state estimation
- Example

Observability test

Definition
A linear system is observable if for any $T > 0$ it is possible to determine the state of the system $x(T)$ through measurements of $y(t)$ and $u(t)$ on the interval $[0, T]$.

Observability test (for linear system)
Consider a linear system of the form

$$\frac{dx}{dt} = Ax + Bu, \quad y = Cx$$

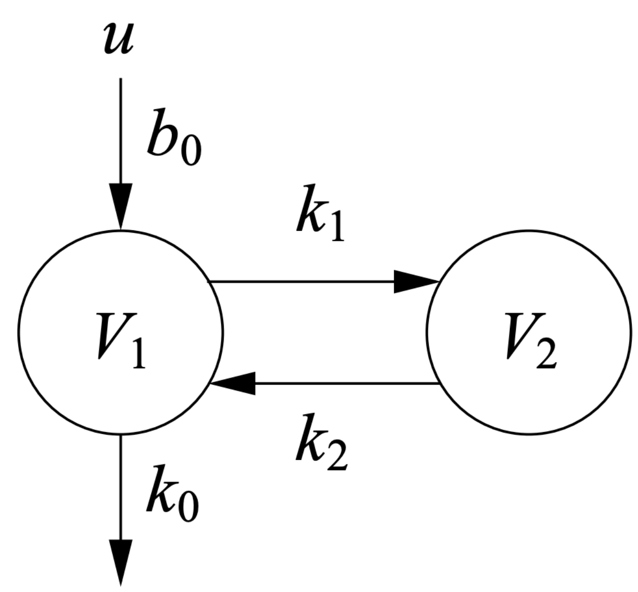
The system is observable if and only if the observability matrix

$$W_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

is invertible. That is, if and only if W_o is of full rank.

Example: Compartment model

drug concentration in blood plasma



drug concentration in tissue

Can we find the concentration in the tissue from measurement of blood plasma?

The two-compartment model is described by

$$\frac{dc}{dt} = \begin{bmatrix} -k_0 - k_1 & k_1 \\ k_2 & -k_2 \end{bmatrix} c + \begin{bmatrix} b_0 \\ 0 \end{bmatrix} u, \quad y = [1 \quad 0]\, x$$

Observability matrix

$$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -k_0 - k_1 & k_1 \end{bmatrix}$$

Concentration in tissue can be determined as long as $k_1 \neq 0$