Lecture 10

- how to prove stability for linear models? -

Stability of linear systems

Theorem
Consider the system

$$\frac{dx}{dt} = Ax$$

The equilibrium $x_e = 0$ is asymptotically stable if and only if all eigenvalues of A have a strictly negative real part.

The equilibrium $x_e = 0$ is unstable if any eigenvalue of A has a strictly positive real part.

Stability of linear systems (discrete-time version)

Theorem
Consider the system

$$x_{k+1} = Ax_k$$

The equilibrium $x_e = 0$ is asymptotically stable if and only if all eigenvalues of A satisfy $\|\lambda_i\| < 1$.

The equilibrium $x_e = 0$ is unstable if any eigenvalue $\|\lambda_i\| > 1$.

Remarks

- Linearization of a system is defined only near an equilibrium point
- If the linearization is asymptotically stable, then the equilibrium point x_e is locally asymptotically stable for the full nonlinear system
- Use stability analysis of linearised model to understand stability of nonlinear system