Topics Covered

- Servo position control.
- Proportional-derivative (PD) compensator.
- Designing control according to specifications.

1 Background

1.1 Servo Model

The QUBE Servo 2 voltage-to-position transfer function is

$$P(s) = \frac{\Theta_m(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}, \tag{1}$$

where K and τ can be determined experimentally (Laboratory 4).

1.2 PID Control

The proportional, integral, and derivative control can be expressed mathematically as follows

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt}. \tag{2}$$

The corresponding block diagram is given in Figure 1. The control action is a sum of three terms referred to as proportional (P), integral (I) and derivative (D) control gain. The controller eq. (2) can also be described by the transfer function

$$C(s) = k_p + \frac{k_i}{s} + k_d s. \tag{3}$$

The functionality of the PID controller can be summarized as follows. The proportional term is based on the present error, the integral term depends on past errors, and the derivative term is a prediction of future errors.

The PID controller described by eq. (2) or (3) is an ideal PID controller. However, attempts to implement such a controller may not lead to a good system response for real-world system. The main reason for this is that measured signals always include measurement noise. Therefore, differentiating a measured (noisy) signal will result in large fluctuations, thus will result in large fluctuations in the control signal.

1.3 PV Position Control

The integral term will not be used to control the servo position. A variation of the classic PD control will be used: the proportional-velocity control illustrated in Figure 2. Unlike the standard PD, only the negative velocity is fed back (i.e. not the velocity of the *error*) and a low-pass filter will be used in-line with the derivative term to suppress measurement noise. The combination of a first order low-pass filter and the derivative term results in a high-pass filter $H(s)$ which will be used instead of a direct derivative.

[1]This laboratory take some material from the Quanser Inc workbooks. And this laboratory represent the 4% of the student's final grade.

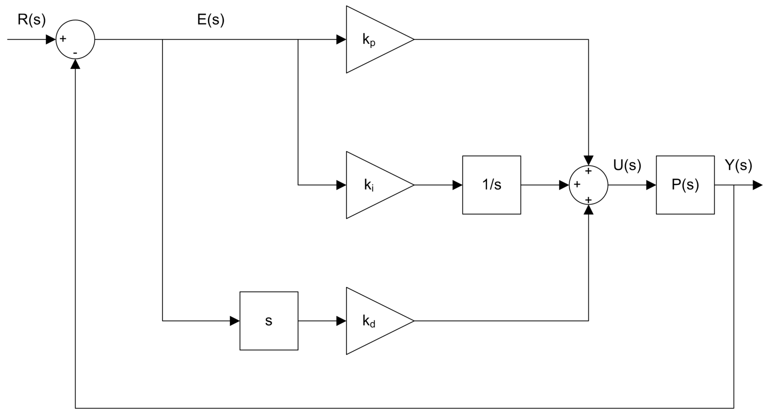

Figure 1: Block diagram of PID control

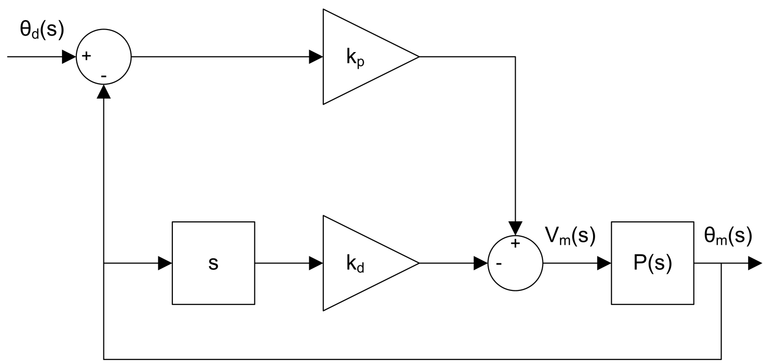

Figure 2: Block diagram of PV control

The proportional-velocity (PV) control has the following structure

$$u = k_p \left(r(t) - y(t)\right) - k_d \dot{y}(t), \tag{4}$$

where k_p is the proportional gain, k_d is the derivative (velocity) gain, $r = \theta_d(t)$ is the setpoint or reference motor / load angle, $y = \theta_m(t)$ is the measured load shaft angle, and $u = V_m(t)$ is the control input (applied motor voltage).

The closed-loop transfer function of the QUBE Servo 2 is denoted $Y(s)/R(s) = \Theta_m(s)/\Theta_d(s)$. Assume all initial conditions are zero, i.e. $\theta_m(0^-) = 0$ and $\dot{\theta_m}(0^-) = 0$, taking the Laplace transform of eq. (4) yields

$$U(s) = k_p(R(s) - Y(s)) - k_d s Y(s),$$

which can be substituted into eq. (1) to result in

$$Y(s) = \frac{K}{s\left(\tau s + 1\right)} \left(k_p \left(R(s) - Y(s)\right) - k_d s Y(s)\right).$$

Solving for $Y(s)/R(s)$, we obtain the closed-loop expression

$$\frac{Y(s)}{R(s)} = \frac{K k_p}{\tau s^2 + \left(1 + K k_d\right) s + K k_p}. \tag{5}$$

This is a second-order transfer function. Recall the standard second-order transfer function

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \tag{6}$$

2 Lab

Design a Simulink model of the PV controller outlined in subsection 1.3. It is is recommended to implement a low-pass filter ($50/(s+50)$) with the derivative term to suppress measurement noise. And perform the following analysis

1. (0.5 points) Set $k_p = 2.5V/rad$ and $k_d = 0V/(rad/s)$. Keep the derivative gain at 0 and vary k_p between 1 and 4. What does the proportional gain do when controlling servo position?

2. (0.5 point) Set $k_p = 2.5V/rad$ and vary the derivative gain k_d between 0 and $0.15V/(rad/s)$. What is its effect on the position response?

3. (0.5 points) Find the proportional and derivative gains required for the QUBE Servo 2 closed-loop transfer function given in eq. (5) to match the standard second-order system in 6. Your gain equations will be a function of ω_n and ζ.

4. (0.5 points) Design a controller that satisfy the following performance specifications:

   - Settling time $< 0.17s$
   - Overshoot $< 2.5\%$

   Attach the position response, as well as the motor voltage in simulation.

5. (1.0 point) Using Arduino, implement your controller. **Assume that you can not measure the angular velocity**. The Arduino must only read the position of the servo motor. Measure the settling time and percent overshoot of the QUBE Servo 2 response and compare that with your expect results. Why does the QUBE Servo 2 response have a steady state error, while the QUBE Servo 2 model response from the transfer function have none? Attach the QUBE Servo 2 position response, as well as the motor voltage.

6. (0.5 points) If your response did not match the above overshoot and settling time specification, try tuning your control gains until your response does satisfy them. Attach the resulting MATLAB figure of the position response, resulting measurements, and comment on how you modified your controller to arrive at those results.

7. (1.0 point) Oral presentation.

Make sure your report is not too long, but concise. Its length should not exceed 5 pages. You must present some conclusions and send the Matlab and Simulink code (0.5 points).