Objective

1. To learn how to use Mathematica to run systems analysis.

2. To understand some of the basic concepts behind the control theory.

Problem 1 - Insulin-Glucose dynamics

The concept of *feedback* is present in many dynamical systems. One of the most famous example is the regulation of the glucose in the blood-stream. The glucose concentration is influenced by different factors (see Fig. 1 [1]). However, the body attemps to maintain a constant concentration of glucose through the production of insulin and glucagon. When glucose levels rice, the pancreas secretes insulin and causes the body to store excess glucose in the liver. When glucose levels are low, the hormone glucagon is produced by the pancreas, which has the opposite effect [1]. To have a better understanding read section 3.6 of "Feedback Systems: An Introduction for Scientist and Engineers" and watch this video (`https://www.youtube.com/watch?v=ae_jC4FDOUc`).
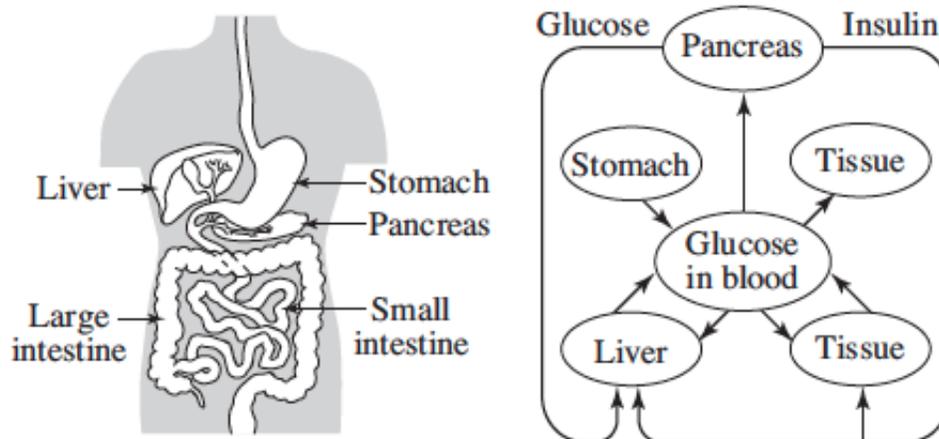

Figure 1: Insulin-glucose dynamics

According to the model suggested by the lecture:

- [0.5 points] Illustrate in a block diagram the system that regulates the levels of glucose in the human body.

- [0.5 point] Using Mathematica, simulate the model system and show and explain the evolution of the concentration of glucose and interstitial insulin. Based on the data presented in [2], consider:

   (a) $p_1 = 0.0264$, $p_2 = 0.0254$, and $p_3 = 0.000012$
   (b) The equilibrium value of glucose as $90 mg/dl$.

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[1]Taken from `http://www.cds.caltech.edu/~murray/books/AM08/pdf/am08-complete_28Sep12.pdf`

(c) The equilibrium value of insulin as $7uU/ml$

(d) The initial values of glucose as $280mg/dl$.

- [0.5 points] Plot the phase-diagram of the system around the point $(90, 0.0)$ (use the function *StreamPlot* in Mathematica). Assume $u = i_e$. What can you affirm about this plot?

- [1.0 point] Diabetes is a disease which affects many people around the world. Which values should you change in the model to simulate the diabetes? Your answer have to be supported through simulations. Using the new model, how is the behavior of the level of glucose if you increase the input (level of insulin in the bloodstream)? Show simulations.

Problem 2 - Mass-Spring system

Consider the following Mass-Spring system shown in Fig. 2. Where $F_s(x)$ is the spring force, $F_f(\dot{x})$ is the friction coefficient, $x(t)$ is the displacement and $F_a(t)$ is the applied force:
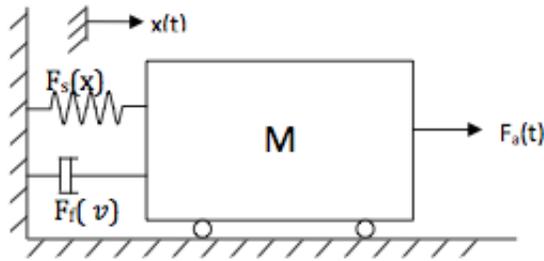

Figure 2: Mass-Spring System

Note that:
Remember that if the displacement of the mass is $x(t)$, the speed is

$$v = \frac{dx(t)}{dt}$$

and the acceleration is

$$a = \frac{dv(t)}{dt}$$

Consider

$$F_f(v) = Bv$$
$$F_s(x) = Kx(t)$$

where B is called the friction coefficient and K is called the spring constant.

- [0.5 points] Using Newton's Law, obtain the dynamic equations of the Mass-Spring system model showed in Figure 2. Consider:

(a) The friction coefficient as $30Nsec/m$.

(b) The spring constant as $15N/m$.

(c) The mass as $700Kg$.

(d) The force a as the input $300N$.

Present the detailed procedure used to find the mathematical model.

- [0.5 points] Based on the model obtained, describe the dynamics of the system in its state-space form. Make sure to consider **all components** of a dynamic model.

- [1.0 point] It is easy to verify that the system has one equilibrium point. Is this point stable in the sense of Lyapunov? or unstable? Use the phase-diagram of the system around the equilibrium point to characterize the stability properties of the equilibrium point. Then, demonstrate your result using the Lyapunov stability theorem.

- [0.5 points] Using Mathematica, simulate the model system and consider two different inputs (Step and Sine function). Describe the behavior of the model for each input.

Note: Remember that the report must include:

- Mathematical Model (state space).

- Procedure.

- Answer to all questions.

- Send an email with the Notebook that you used.

- Conclusions.

- References.

References

1. Karl Johan Astrom, and Richard M. Murray. Feedback Systems: An Introduction for Scientist and Engineer. 2nd Edition. 2012, Princeton University Press.

2. D.R. Kerner. Minimal Models for Glucose and Insulin Kinetics.