Lecture 9
- rationalizable strategies -

Review

- An epistemic battle of the sexes
- Nash Equilibrium in conjecture
- Bayesian Rationality

An epistemic battle of the sexes

- Types of players:
  - $A_1,\ A_2,\ A_3,\ A_4$
  - $V_1,\ V_2,\ V_3,\ V_4$

Violetta

Alfredo		g	o
	g	2,1	0,0
	o	0,0	1,2

- State of the game, for $i,j = 1,\ldots,4$

$$\omega_{ij} = (\omega_{ij}^A, \omega_{ij}^V, \omega_{ij}^s, \omega_{ij}^t)$$

Alfredo's type is A_i
and Violetta's type is V_i

Alfredo plays s_i
and Violetta plays t_i

Nash in conjecture

	$\mathbf{s}_i(\omega)$	ϕ_i^ω
A_1	$s_1 = \text{o}$	o
A_2	$s_2 = \text{g}$	g
A_3	$s_3 = \text{g}$	*
A_4	$s_4 = \text{o}$	*

	$\mathbf{s}_i(\omega)$	ϕ_i^ω
V_1	$t_1 = \text{o}$	o
V_2	$t_2 = \text{g}$	g
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$*$ best mixed strategy response

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Nash in conjecture:
each player's conjecture is a best response to the other player's conjecture

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Nash in conjecture:
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Common priors,
mutual knowledge of rationality
$\rightarrow$ Nash in conjecture

Bayesian Rationality

$$\pi_i(s_i, \phi_i^\omega) = \sum_{s_{-i} \in S_{-i}} \phi_i^\omega(s_{-i}) \pi(s_i, s_{-i})$$

Bayesian Rationality

- If Player i pure strategy choice $s_i = \mathbf{s}_i(\omega) \in S_i$ maximizes

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- If for every state $\omega \in \Omega$

$$\pi_i(\mathbf{s}_i(\omega), \phi_i^\omega) \geq \pi_i(s_i, \phi_i^\omega)$$

all $s_i \in S_i$

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all $s_i \in S_i$

A player is rational at state ω if his pure strategy $\mathbf{s}_i(\omega)$
is the best response to his conjecture ϕ_i^{ω}
of the other players' strategies

Bayesian Rationality

- Bayesian
  - uninformed players put probabilities on events
- Rationality:
  - beliefs and actions consistent with each other

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Are the player's actions the best response, given the player's beliefs and the actions and beliefs of the other players?

How are players likely to play?

- Solution concepts
  - dominated strategy
  - iterated dominant strategy
  - Nash Equilibrium

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dominated strategy

strategy s_i' $\in S_i$ is strongly dominated by $s_i \in S_i$ if:

$$\forall \sigma_{-i} \in \Delta^* S_{-i},$$
$$\pi(s_i, \sigma_{-i}) > \pi(s_i', \sigma_{-i})$$

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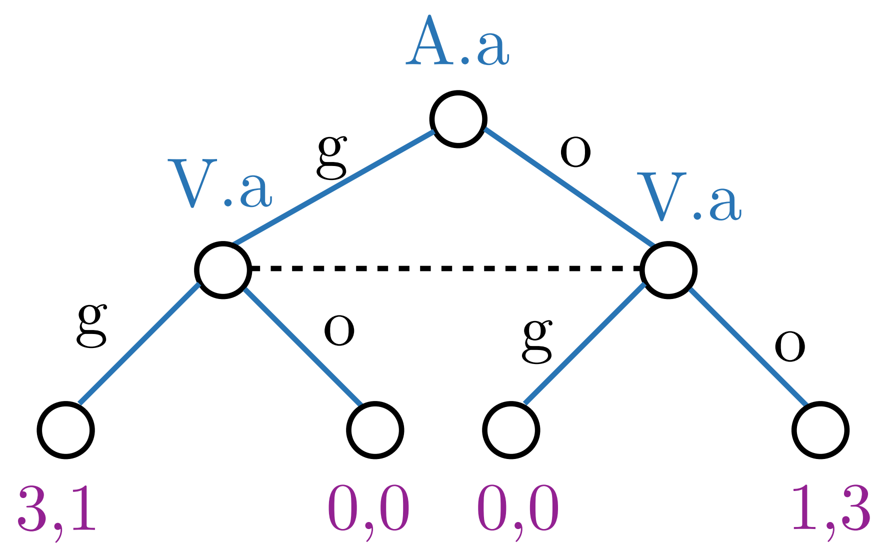
Will rational players eliminate weakly dominated strategies?

Eliminating weakly dominated strategies



Violetta

Alfredo		g	o
	g	3,1	0,0
	o	0,0	1,3

A.0

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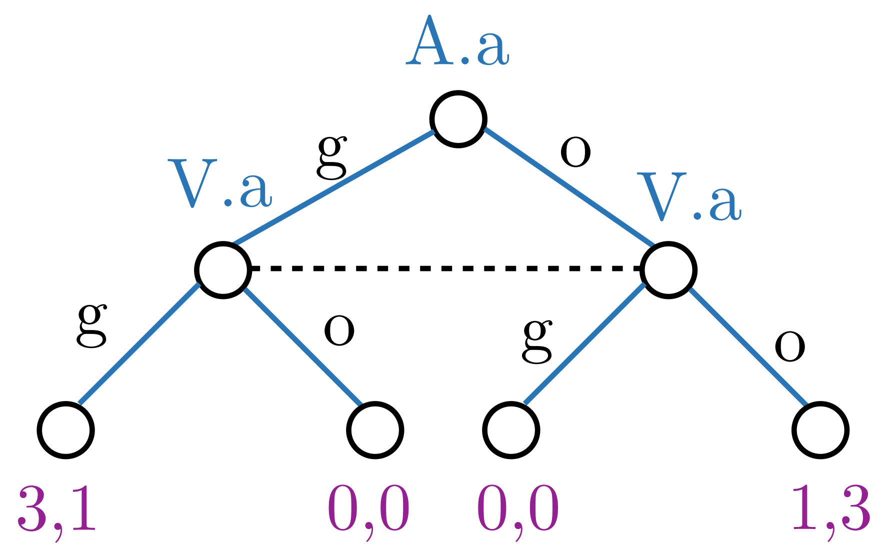
Eliminating weakly dominated strategies



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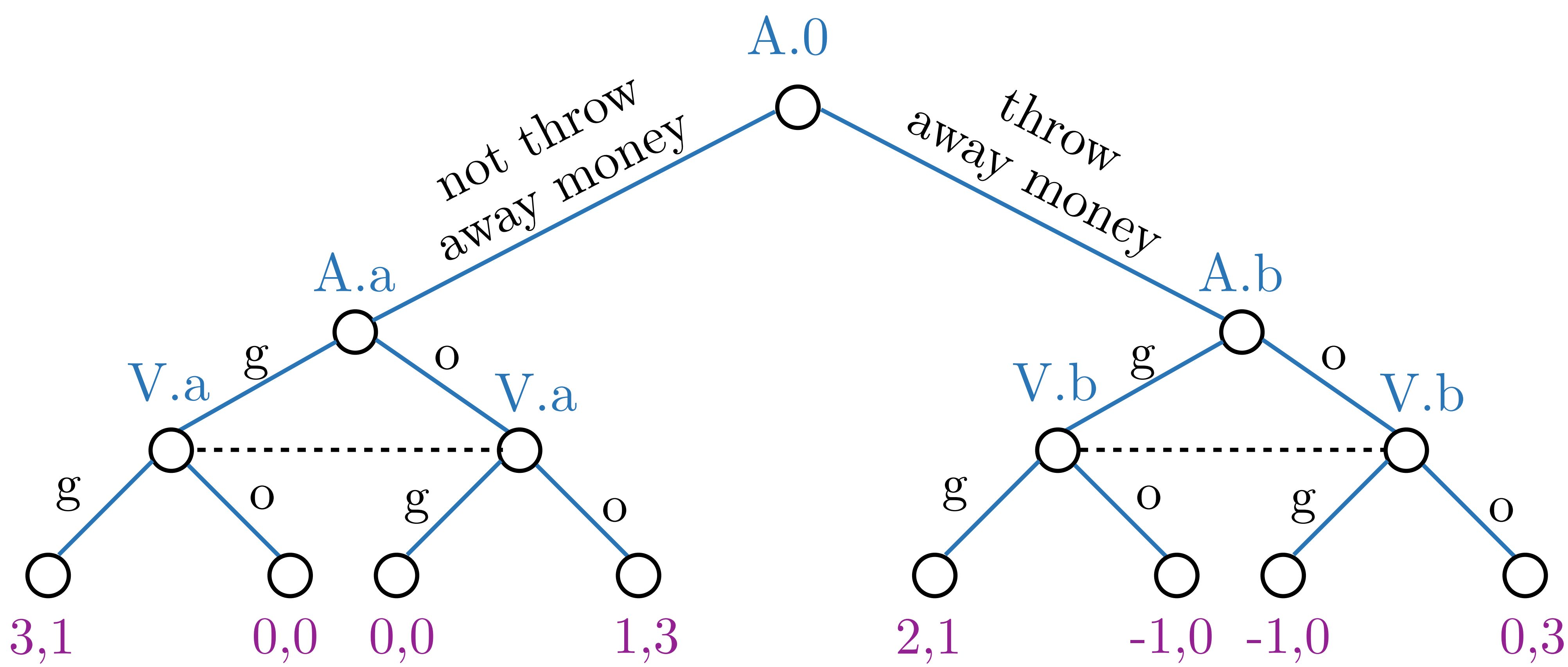
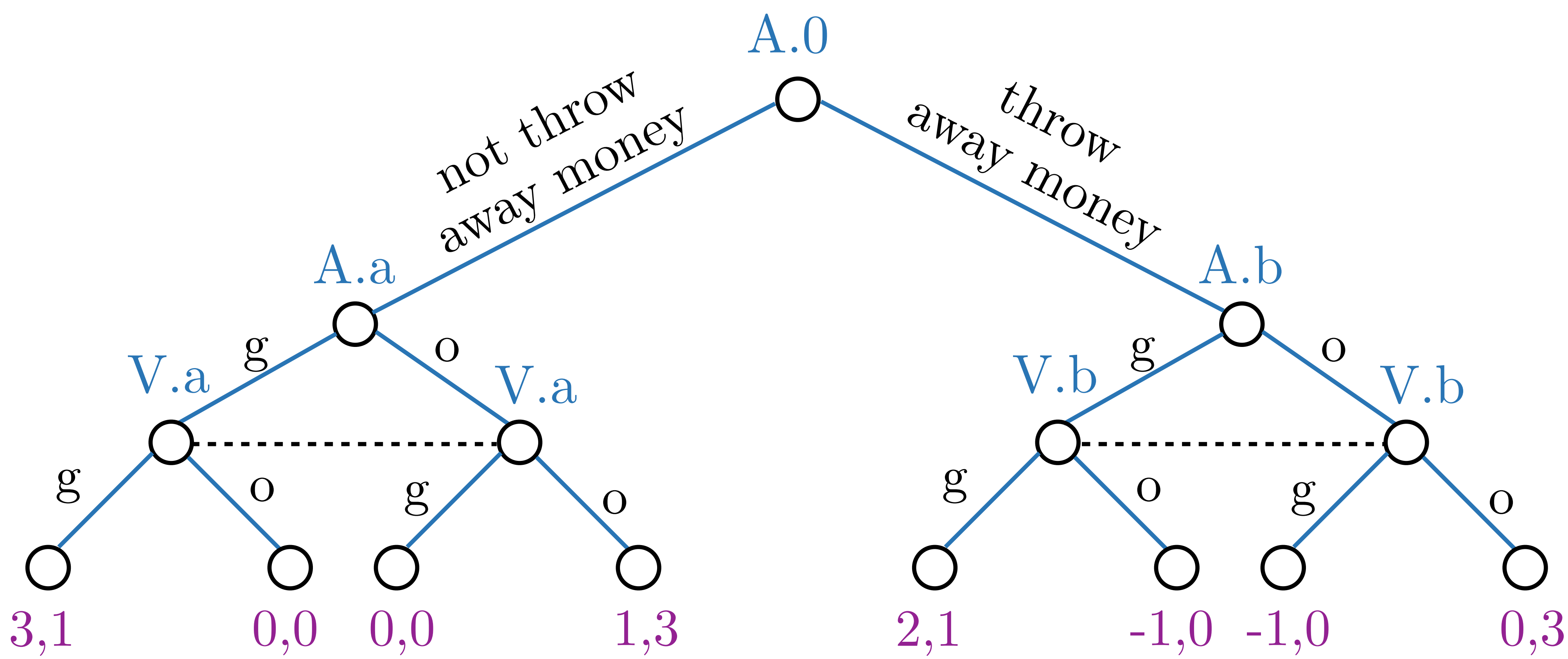


Normal form



Violetta

Alfredo		gg	go	og	oo
	ng	3,1	3,1	0,0	0,0
	no	0,0	0,0	1,3	1,3
	bg	2,1	-1,0	2,1	-1,0
	bo	-1,0	0,3	-1,0	0,3

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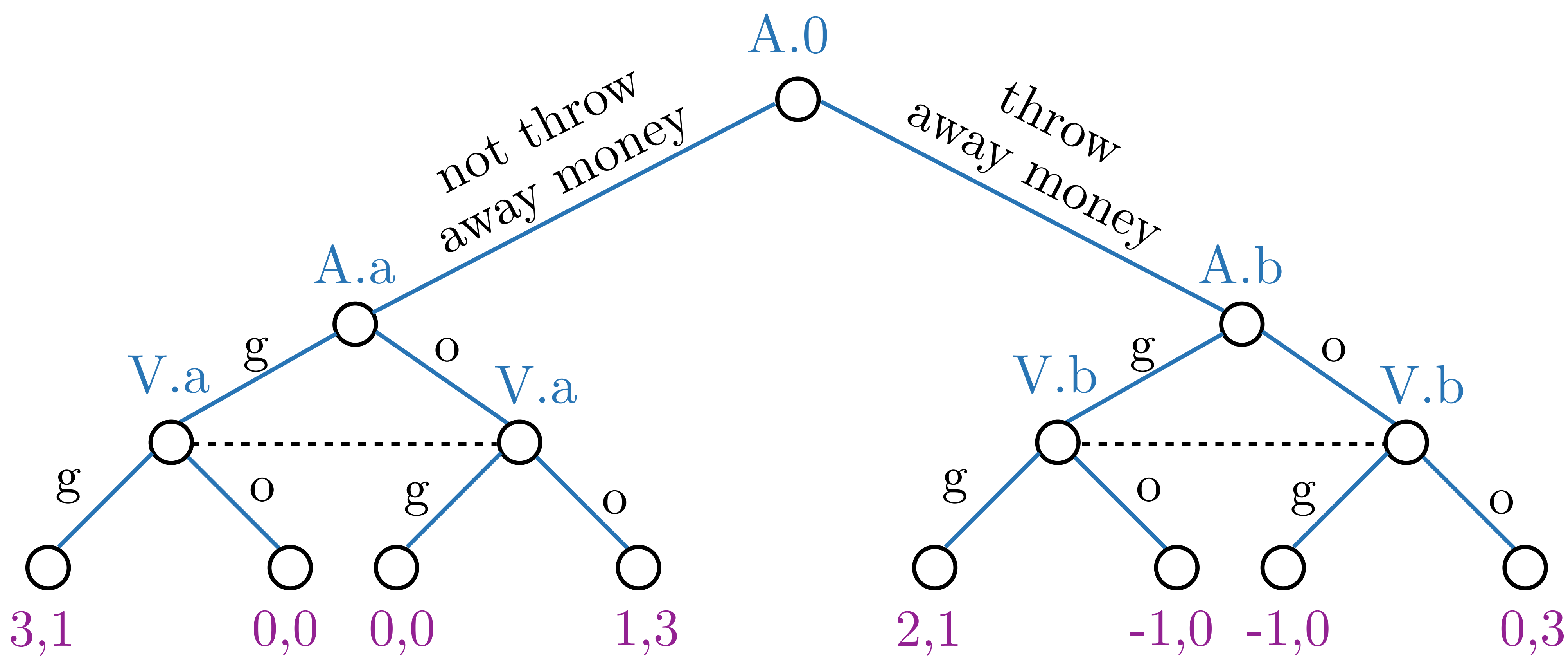

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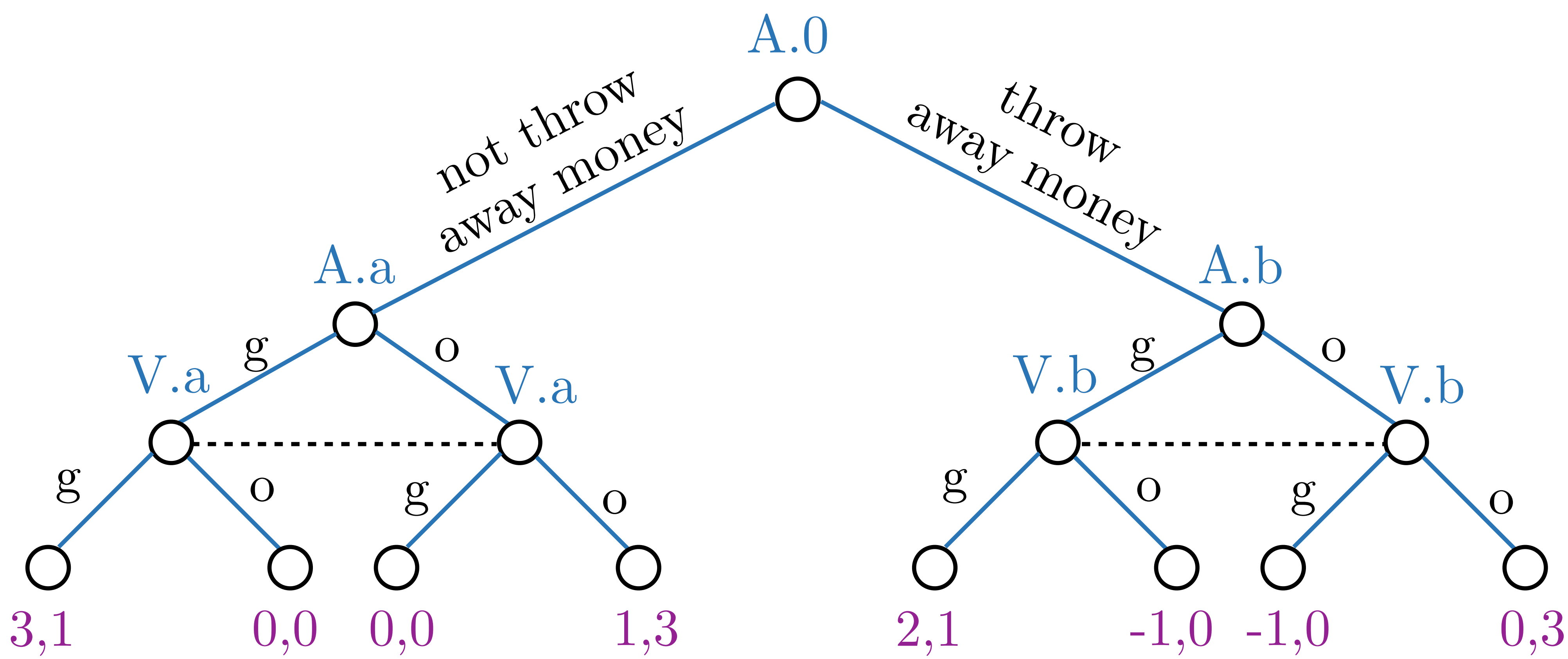

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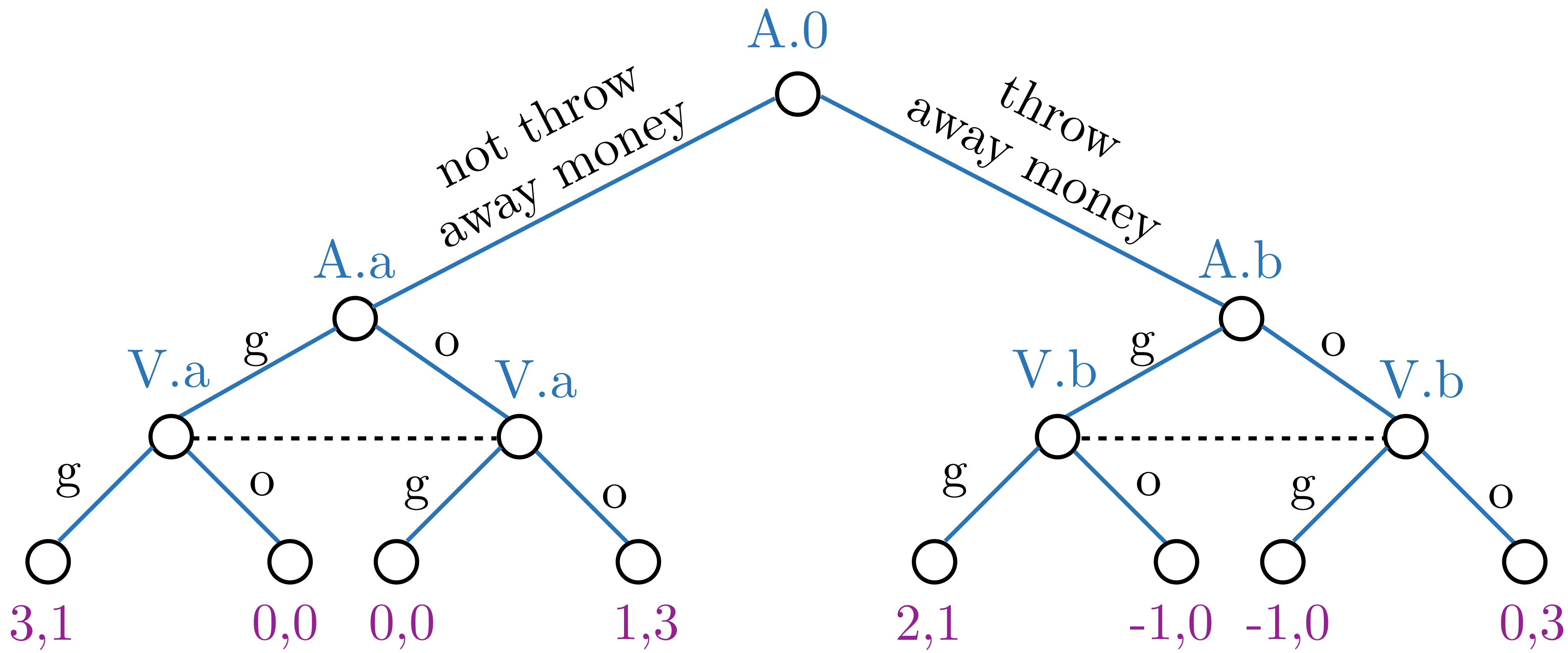

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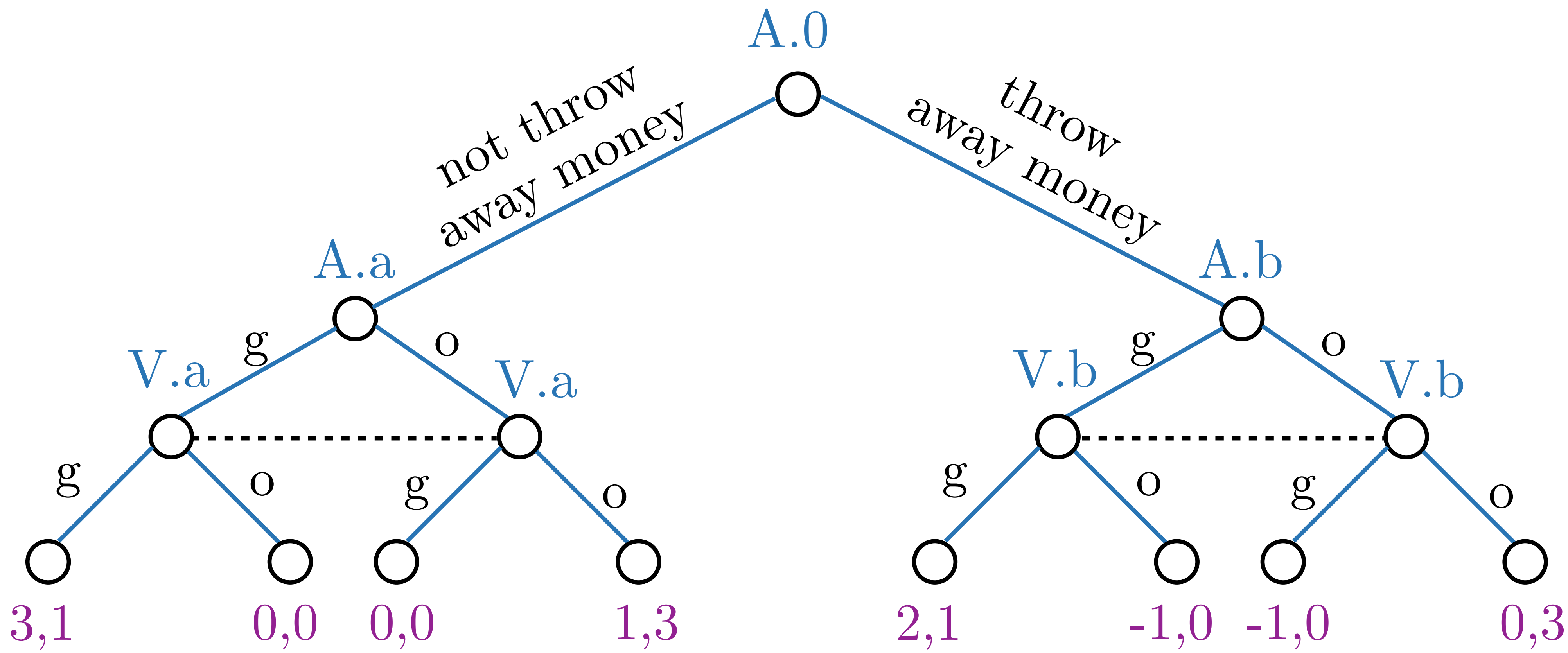

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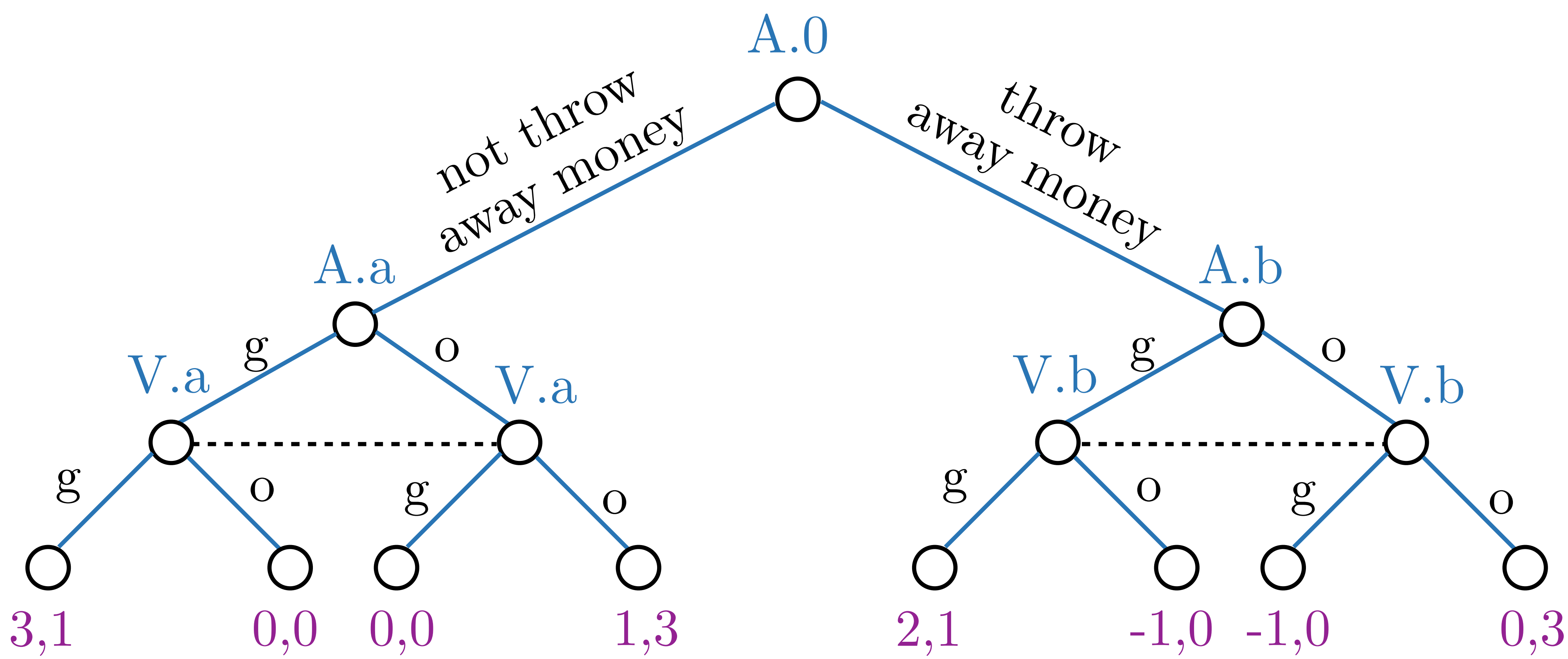

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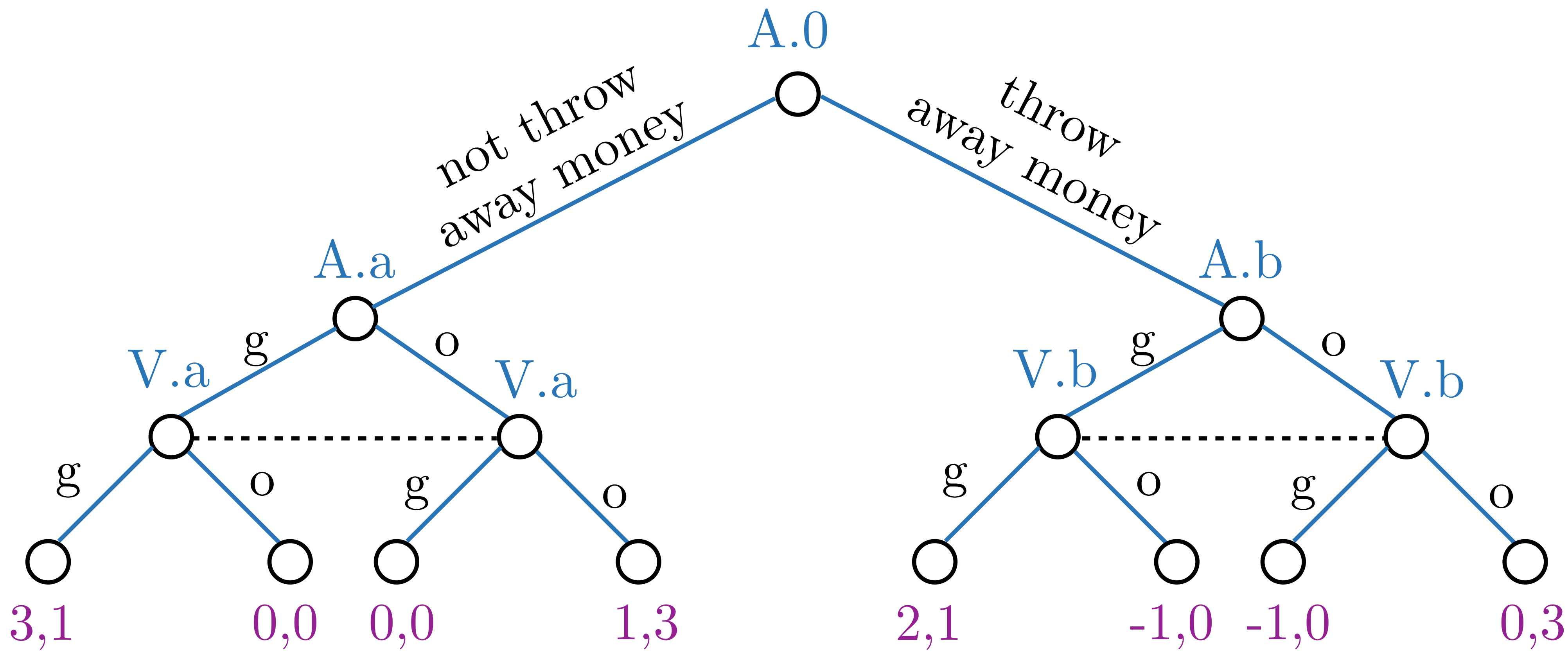

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Throwing away money lead to better outcome for Alfredo?

Today

- Strategy elimination in normal form games
- Rationalizable Strategies
- Best response sets

Eliminating strongly dominated strategies

Player i

		l	c	r
	u	0,2	3,1	2,3
Player j	m	1,4	2,1	4,1
	d	2,1	4,4	3,2

Eliminating strongly dominated strategies

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	~~u~~	~~0,2~~	~~3,1~~	~~2,3~~
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strongly dominated by d

Eliminating strongly dominated strategies

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Eliminating strongly dominated strategies

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strongly dominated by d

Player j		l	c	~~r~~
	m	1,4	2,1	~~4,1~~
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strongly dominated by $0.5\mathrm{l} + 0.5\mathrm{C}$

Eliminating strongly dominated strategies

Player i

Player j

	l	c	r
u	0,2	3,1	2,3
m	1,4	2,1	4,1
d	2,1	4,4	3,2

strongly dominated by d

Player j

	l	c	r
m	1,4	2,1	4,1
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strongly dominated by $0.5l + 0.5C$

Player j

	l	c
m	1,4	2,1
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Eliminating strongly dominated strategies

Player i

Player j		l	c	r
	~~u~~	~~0,2~~	~~3,1~~	~~2,3~~
	m	1,4	2,1	4,1
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strongly dominated by d

Player j		l	c	~~r~~
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Player j		l	c
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Player j		l	c
	~~m~~	~~1,4~~	~~2,1~~
	d	2,1	4,4

strongly dominated by d

Player j		~~l~~	c
	d	~~2,1~~	4,4

strongly dominated by l

Eliminating strongly dominated strategies

Player i

Player j

	l	c	r
u	0,2	3,1	2,3
m	1,4	2,1	4,1
d	2,1	4,4	3,2

strongly dominated by d

Player j

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strongly dominated by $0.5\mathrm{l} + 0.5\mathrm{C}$

Player j

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d	2,1	4,4

strongly dominated by d

Player j

	l	c
d	2,1	4,4

strongly dominated by l

Player j

	c
d	4,4

unique Nash

Matching pennies

- Best response set?

	a	b
a	1, -1	-1, 1
b	-1, 1	1, -1

Matching pennies

- Best response set?
- Completely mixed Nash
  $\rightarrow$ All strategies are rationalizable

	a	b
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Matching pennies

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- Pure strategies s_i with positive probability in a Nash form a best response set, when the players' conjectures the actual mixed strategy choice of the other players

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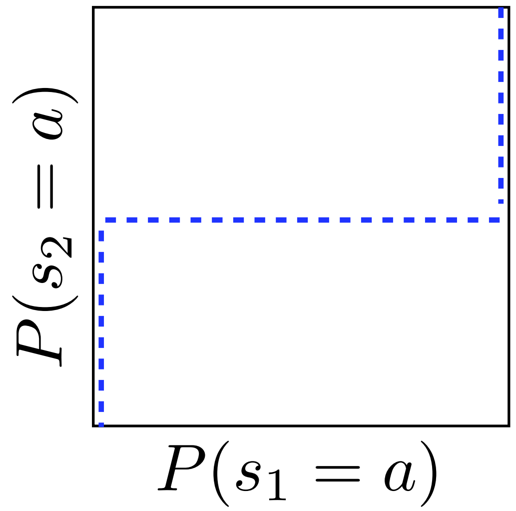
- Best response correspondence for both players



Player 1

Matching pennies

	a	b
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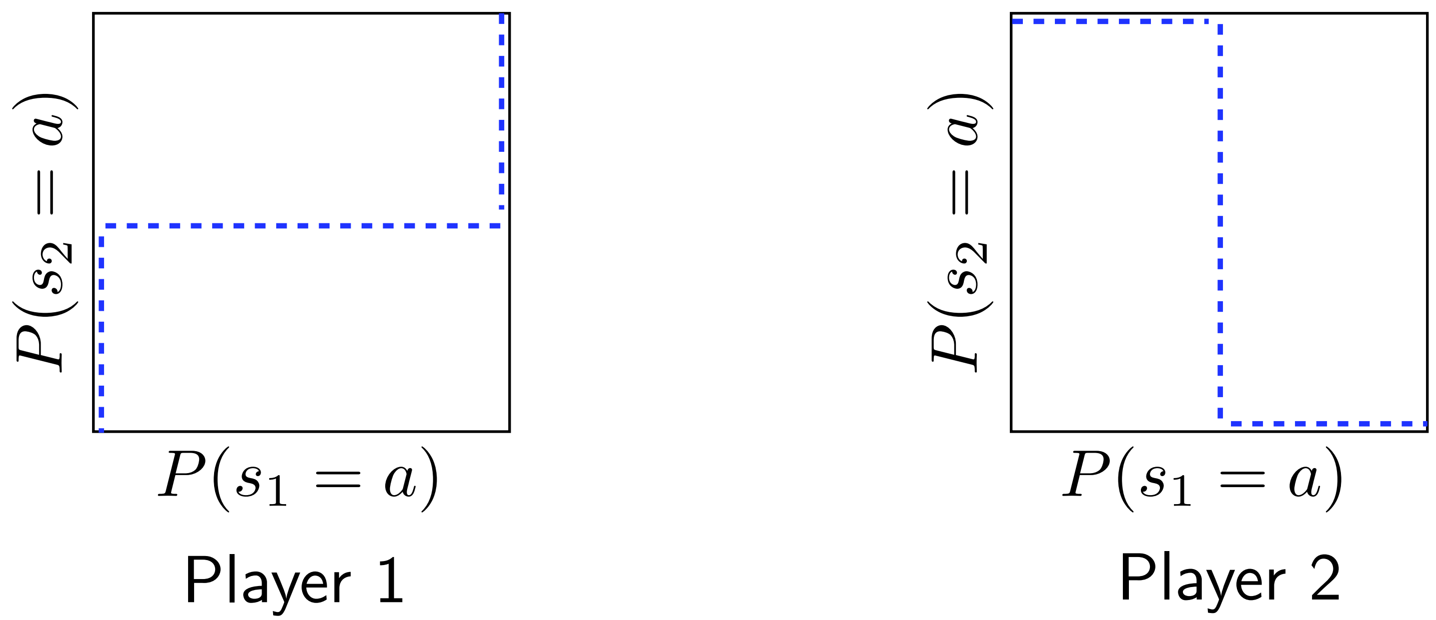

Matching pennies

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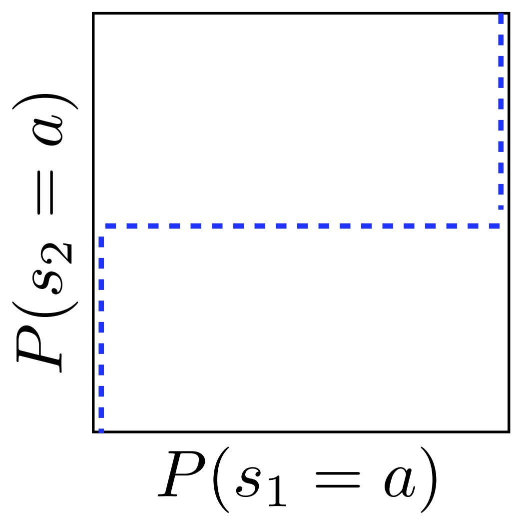
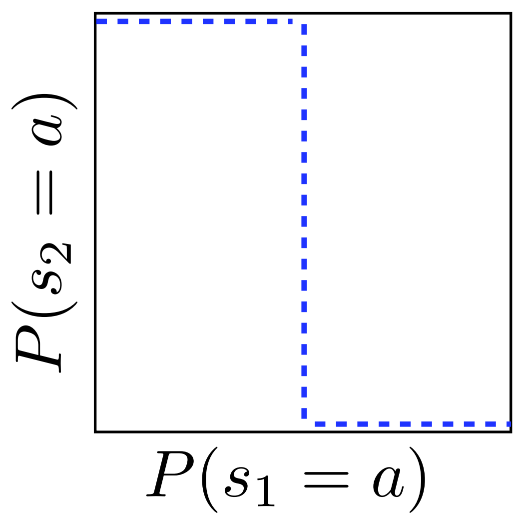
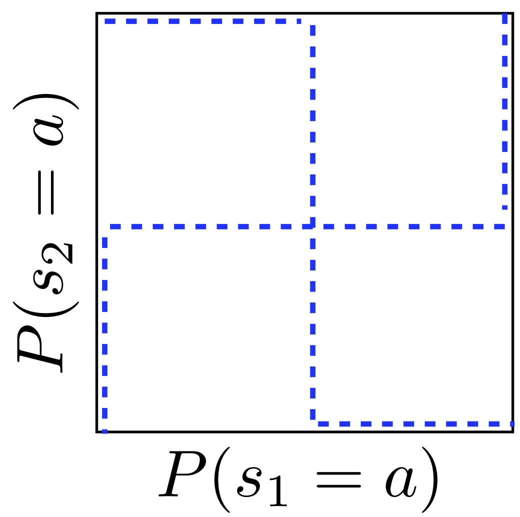


Player 1



Player 2



both players

Next class

- Common Knowledge of rationality
- Extensive form rationalizability
- Strategy elimination in extensive form games
    - Informationally richer
    - Backward induction
        - iterated elimination of weakly dominated strategies
        - subgame perfect Nash (refinement)
    - A dynamic model of market preemption
    - Subgame perfection