Lecture 3
-Bayesian rationality-

Review

- Preference ordering
  - completeness:
    $x \succcurlyeq_A y$ or $y \succcurlyeq_A x$
  - transitivity:
    $x \succcurlyeq_A y$ and $y \succcurlyeq_A z$ implies $x \succcurlyeq_A z$
  - independence of irrelevant alternatives
    For $x, y \in B$, $x \succcurlyeq_B y$ if and only if $x \succcurlyeq_A y$

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consistent

Use math to define a set of properties of the binary relation $\succcurlyeq$ that captures consistent preference ordering!!

Review

Theorem 1.1 - A binary relation $\succcurlyeq$ on a finite set of payoffs X can be represented by a preference function $u\!:\!\mathrm{X}\rightarrow\mathbb{R}$ if and only if $\succcurlyeq$ is consistent.

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Theorem 1.1 - A binary relation $\succcurlyeq$ on a finite set of payoffs X can be represented by a preference function $u{:}\mathrm{X}\rightarrow\mathbb{R}$ if and only if $\succcurlyeq$ is consistent.

Theorem 1.2 - If $u(\cdot)$ represents the preference relation $\succcurlyeq$ and $f(\cdot)$ is a strictly increasing function, then $v(\cdot)=f(u(\cdot))$ also represents $\succcurlyeq$.

Conversely, if both $u(\cdot)$ and $v(\cdot)$ represent $\succcurlyeq$, then there is an increasing function $f(\cdot)$: $v(\cdot)=f(u(\cdot))$.

If an individual prefers x to y and y to z, we can rationalize this behavior by supposing he attaches more utility to x than to y and more utility to y than to z.

Utility function

- Subject prefers x to y and y to z, $\mathrm{X}=\{x,\ y,\ z\}$
- Rationalize - suppose he attaches utilities such that:

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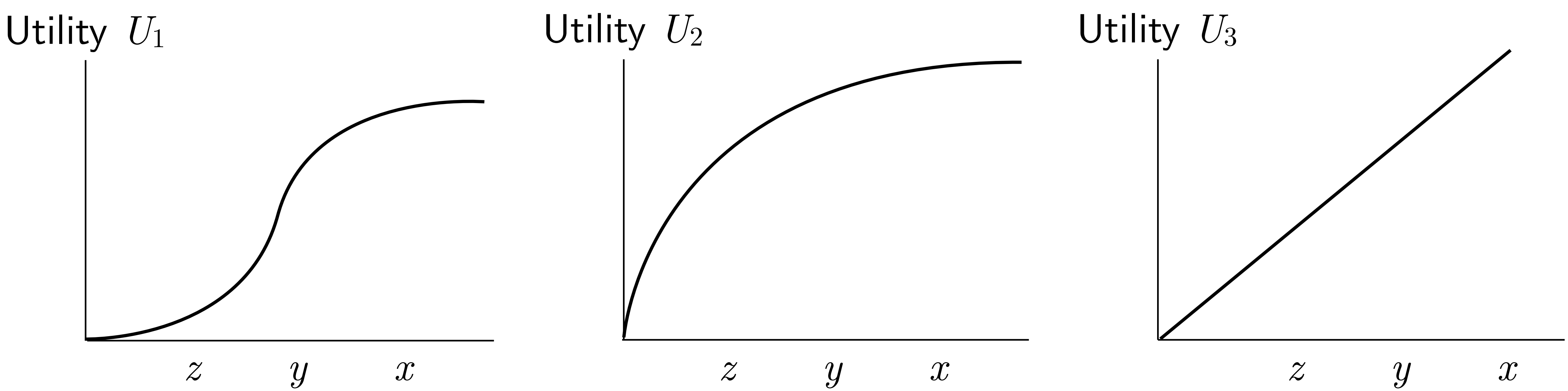
- Common quantitative characteristic
- Behave as if maximizing the total
- Better?



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- Alternatives expressed in terms of money (income)
- Additional information (risky alternatives)
- 50-50 chance of x or z over y

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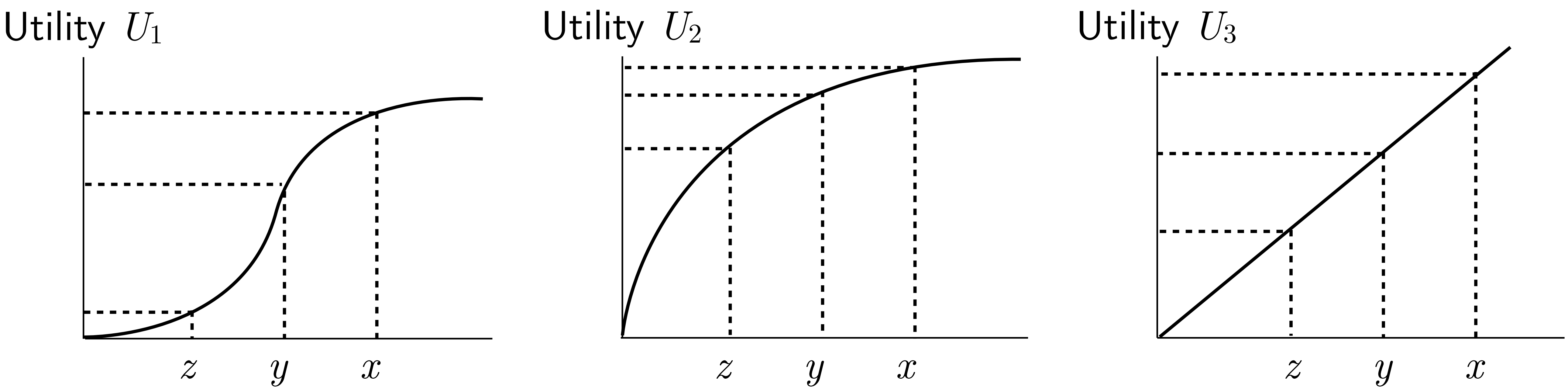

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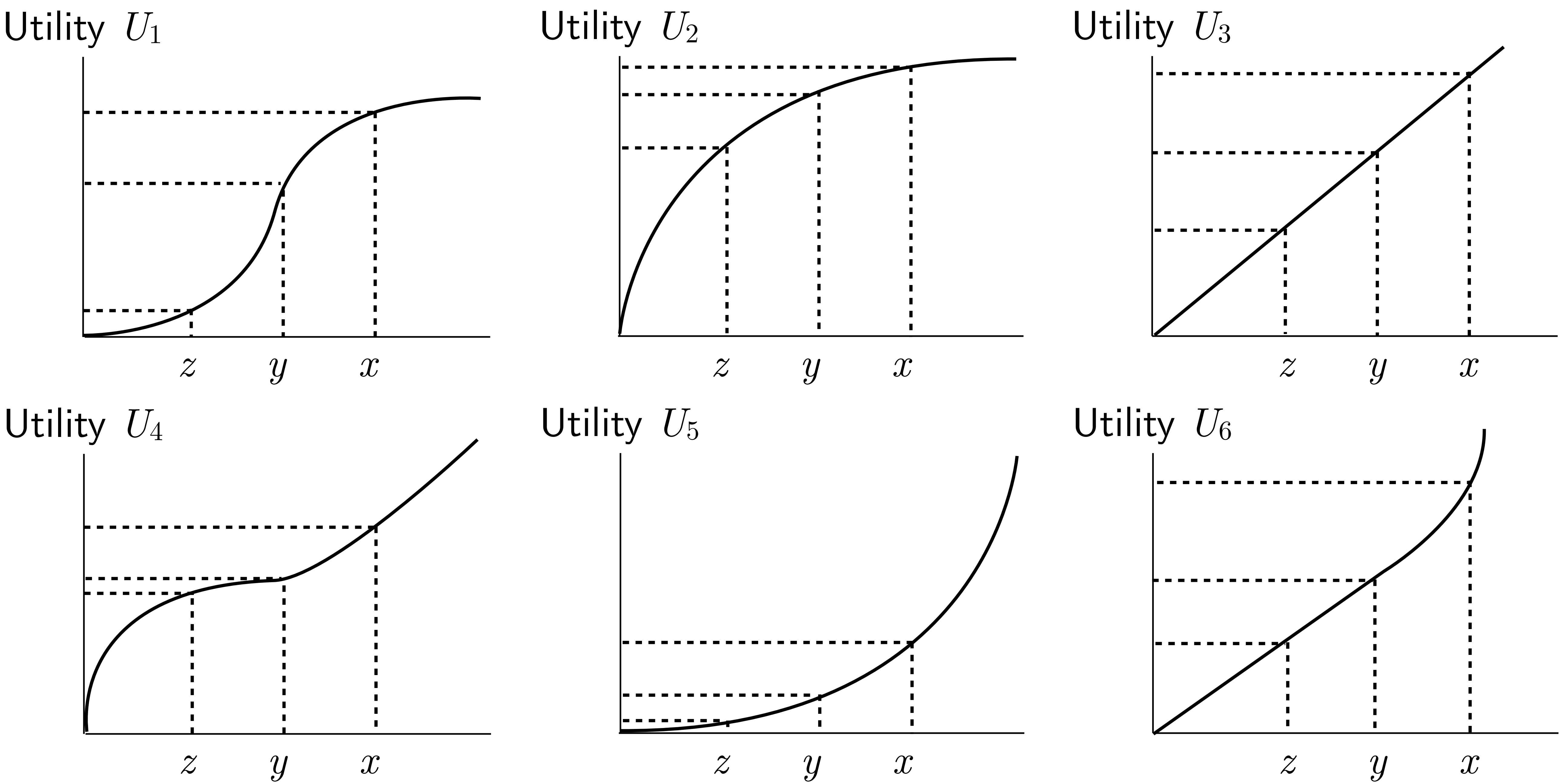

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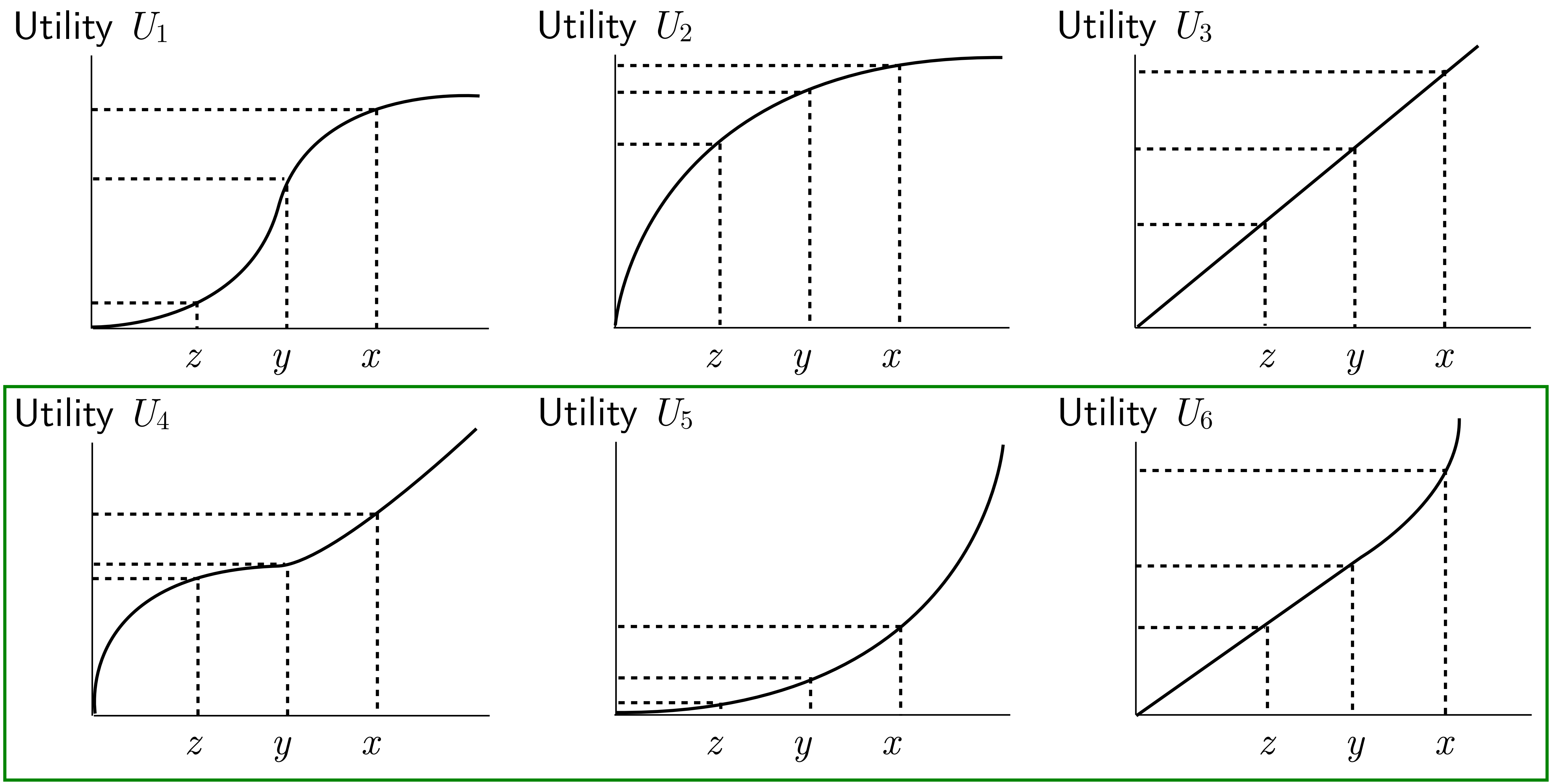

Utility function

- Designated utility
- Expected utility is greater
- Restricted class of functions
- Utility maximization $\nrightarrow$ diminishing marginal utility

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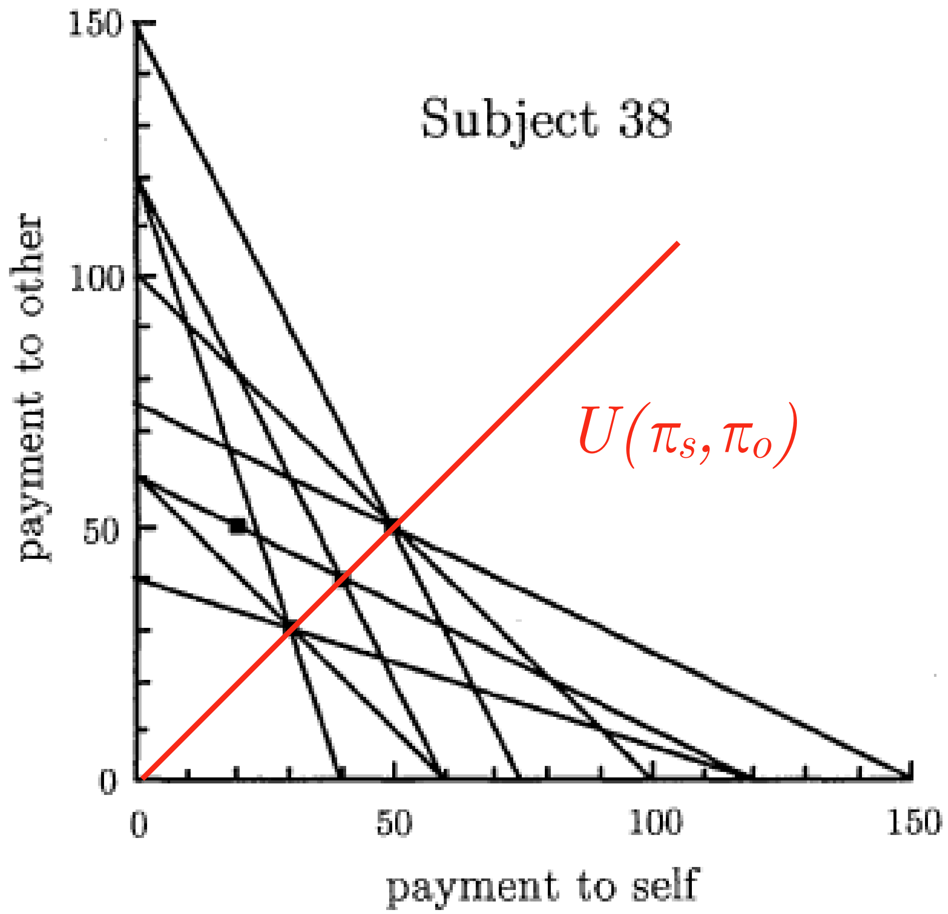
Example

- Leontief:
  $U(\pi_s, \pi_o) = \min\{\pi_s,\ \pi_o\}$



Utility function

- Alternative 1 (gamble or lottery):

  A chance p $(0 < p < 1)$ of x and a chance 1-p of z

- Alternative 2:

  A certain income y

- What would you choose?

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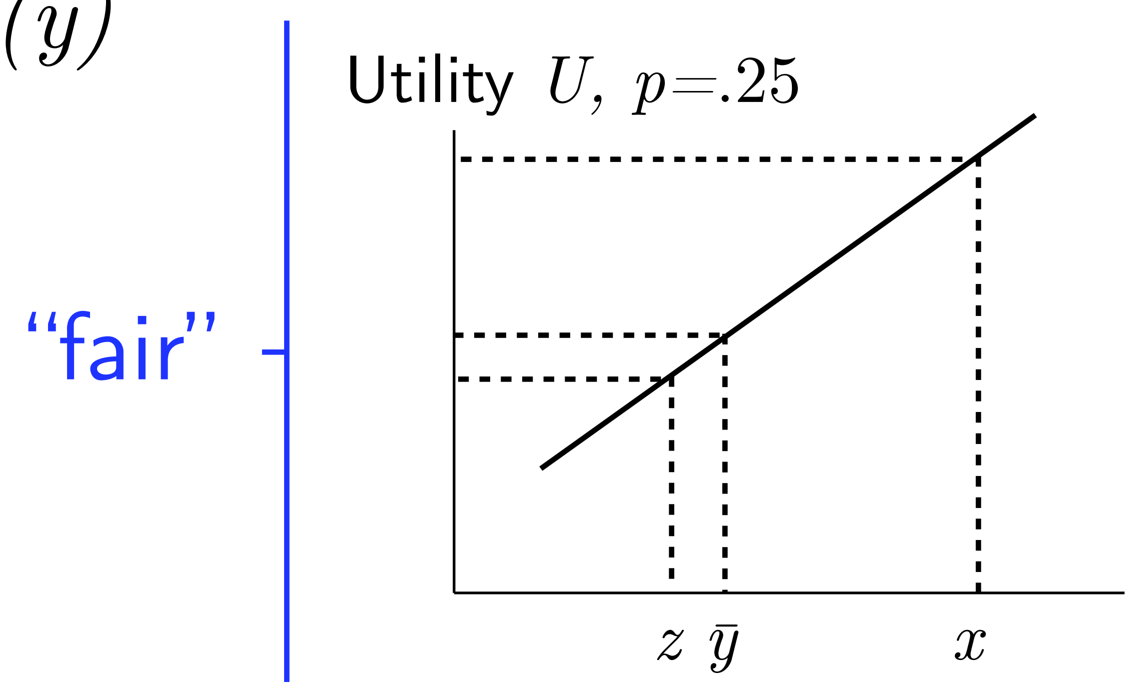
  Utility of Alternative 2: $U_2 = U(y)$

- Fix p. Let $\bar{y}$: $U_2(\bar{y}) = U(\bar{y}) = U_1$
- $y = \bar{y}$: fair gamble
- As p increases, $\bar{y}$ increases



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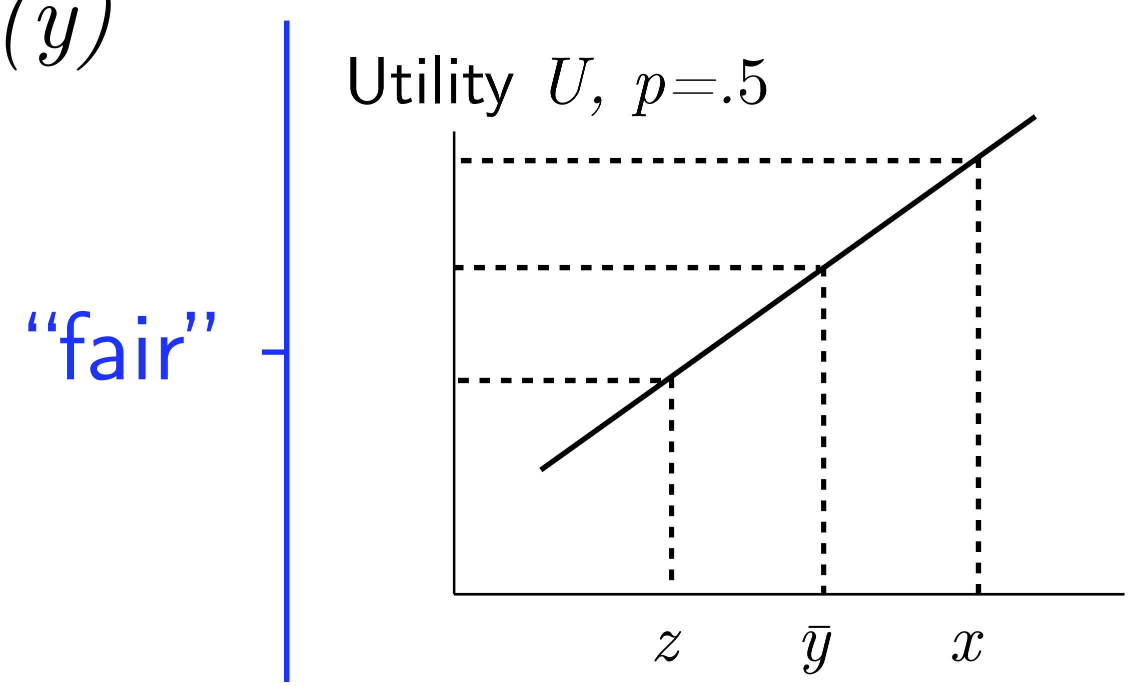

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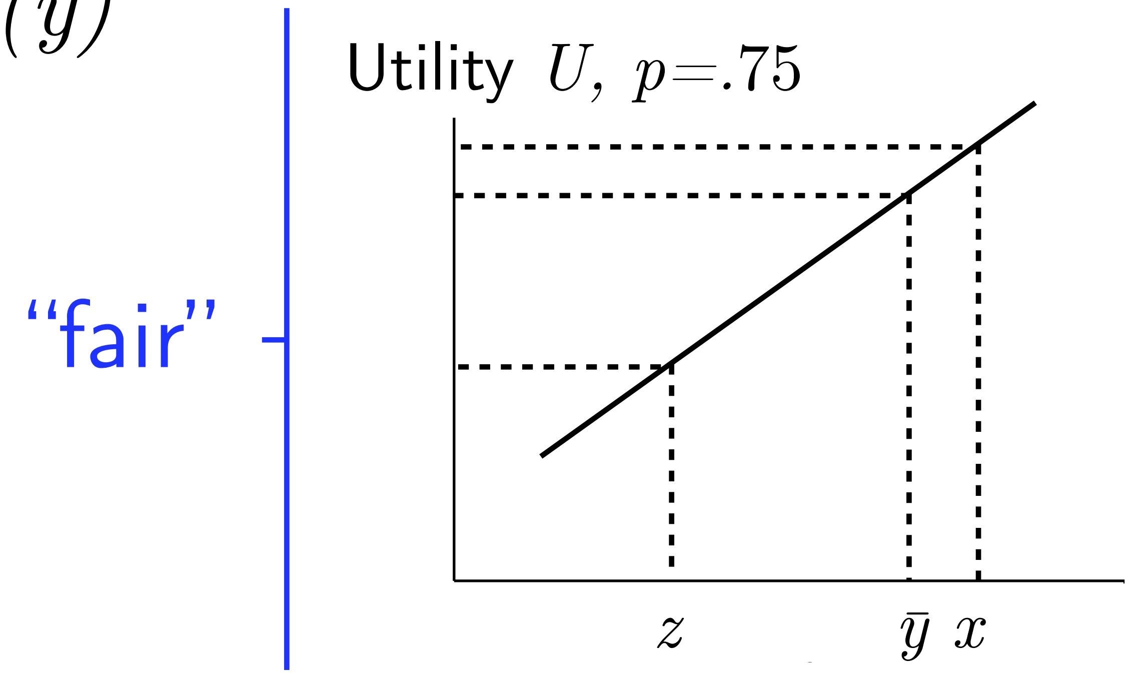

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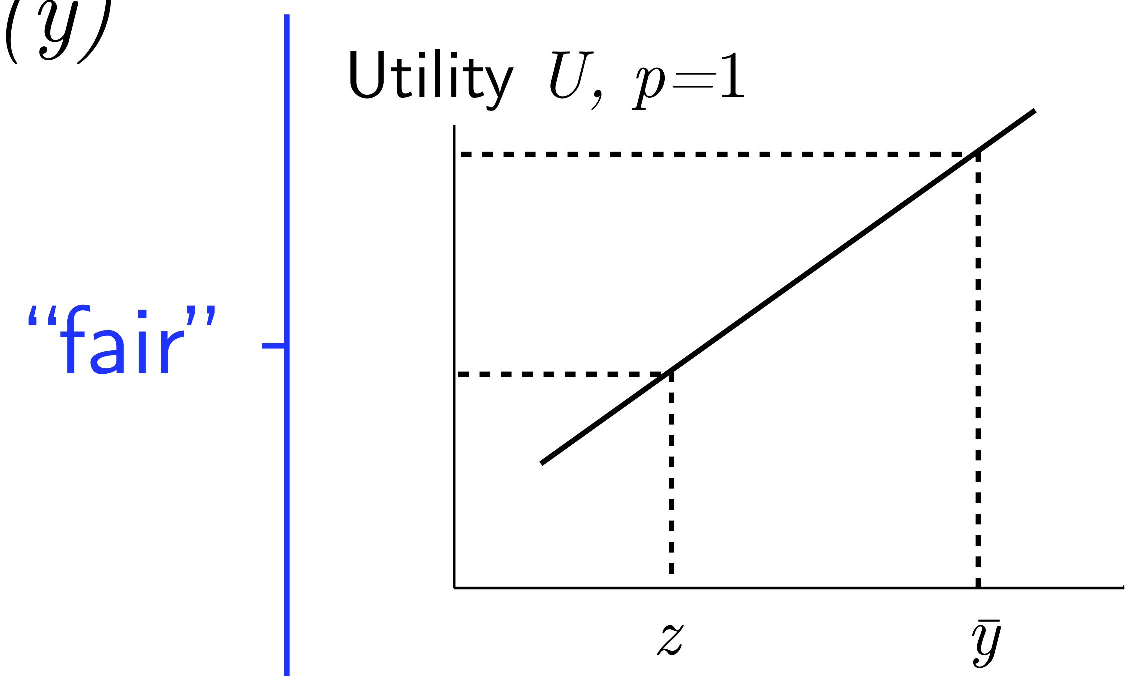
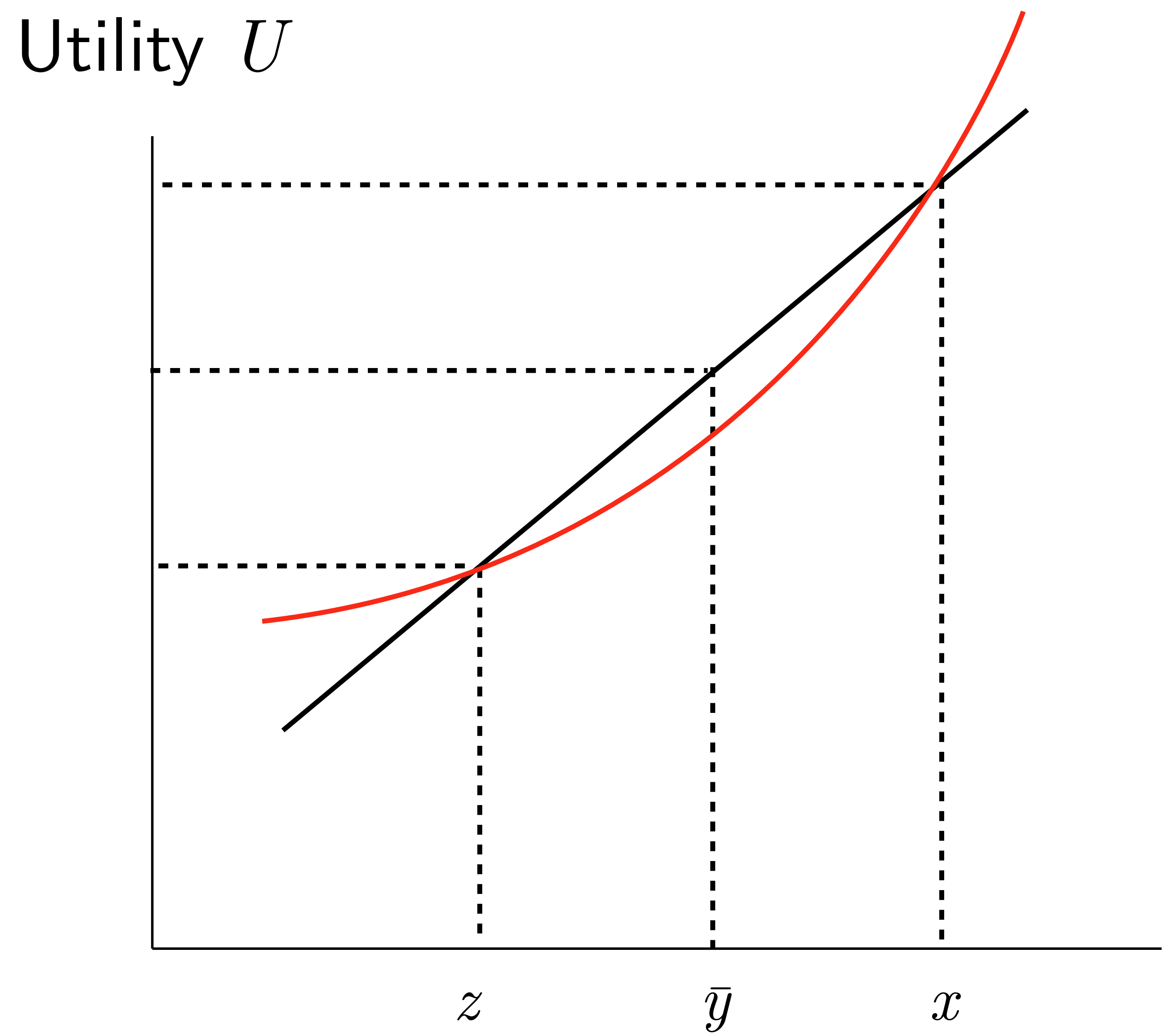
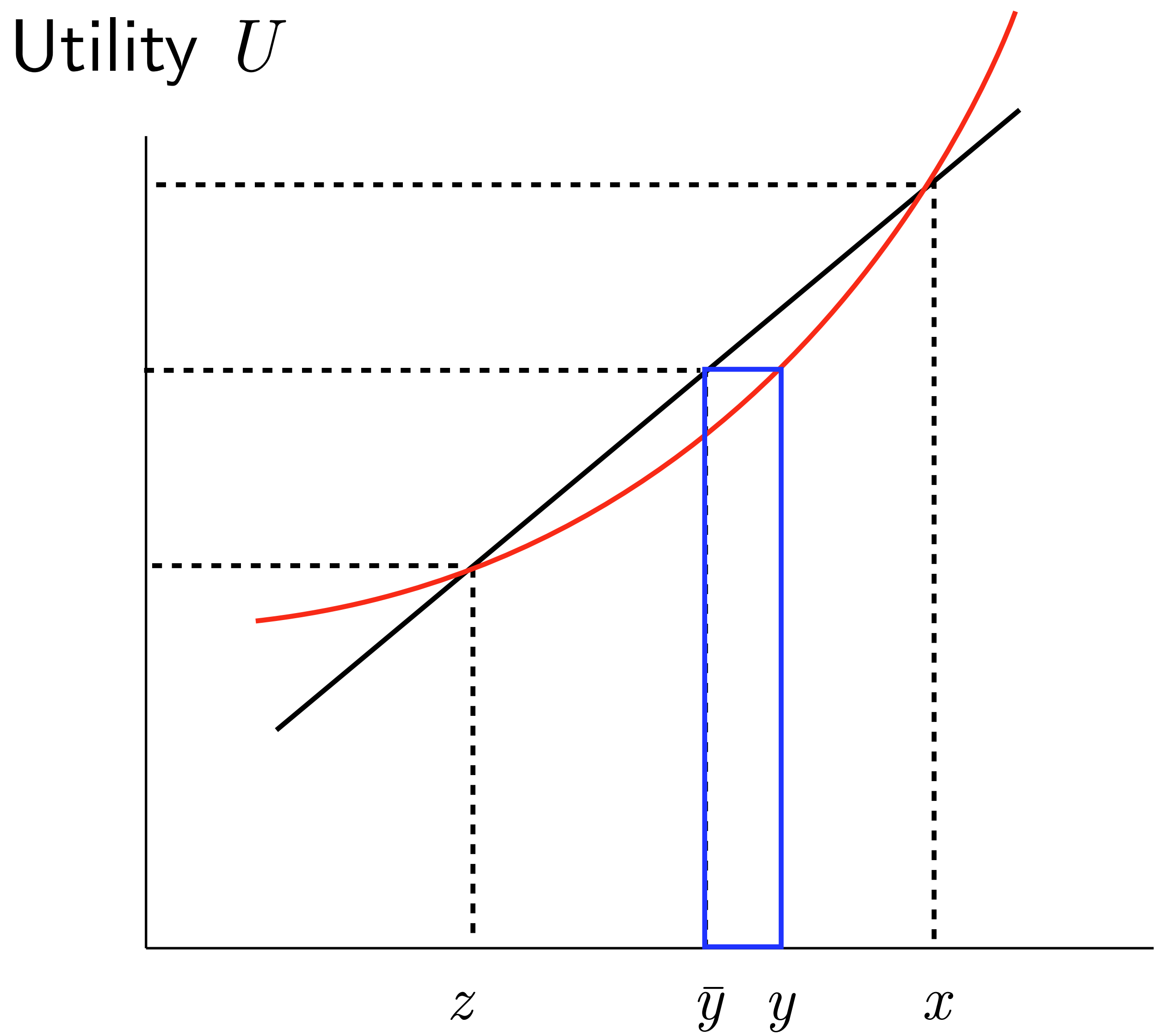




Preference for risk



Preference for risk

Today

- Formalize previous argument
- Expected utility principle

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- Bayesian rationality
    - Savage's Axioms
    - Savage's Theorem

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Next class:

  - Strategic games
  - Representations

Bayesian Rationality

- Stochastic events determine payoffs
- Set of prizes X, lottery with payoffs in X
- Expected value (only one player)

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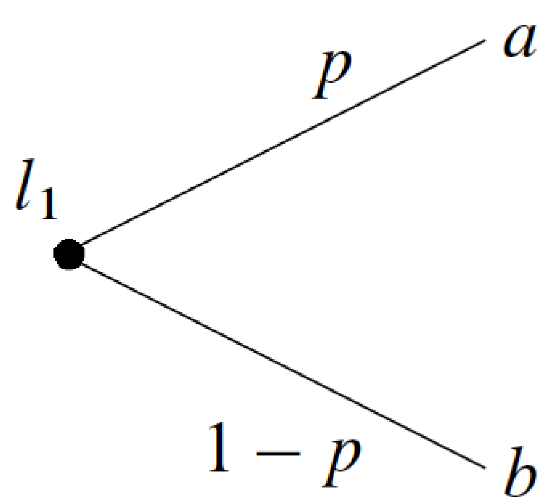

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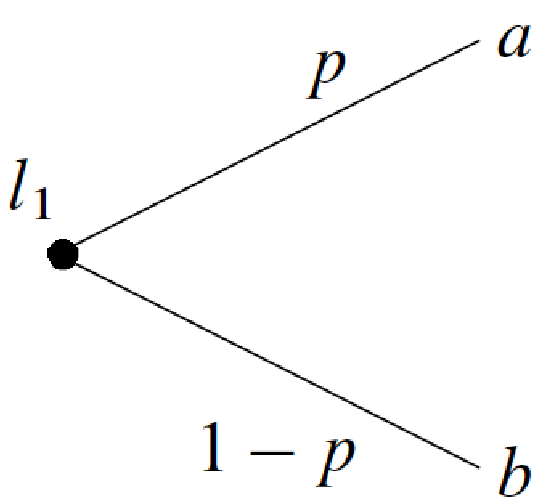

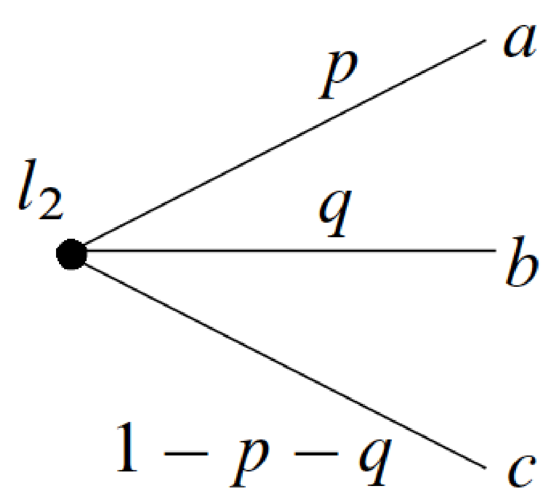

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$$E[l_2] = pa + qb + (1-p-q)c$$

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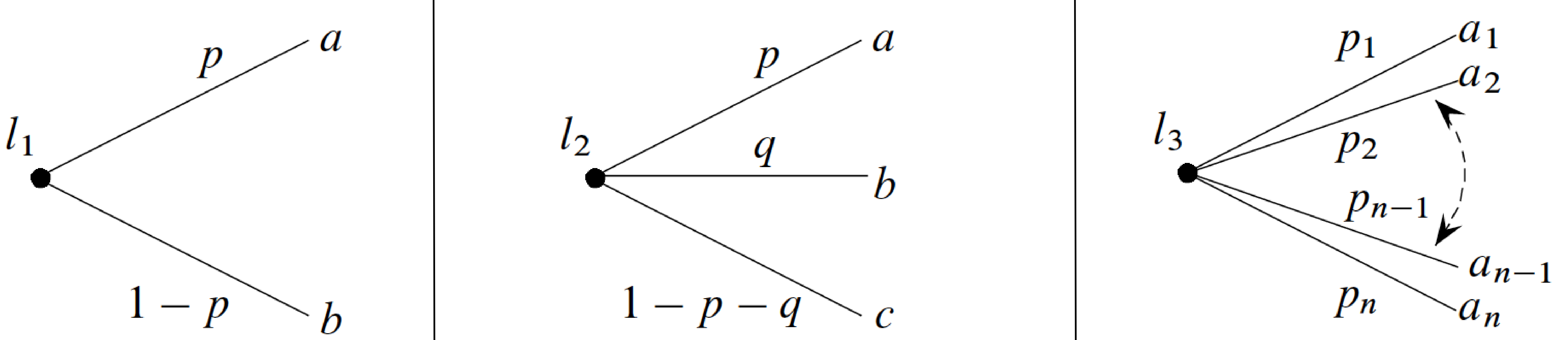

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$E[l_3] = p_1 a_1 + \ldots + p_n a_n$

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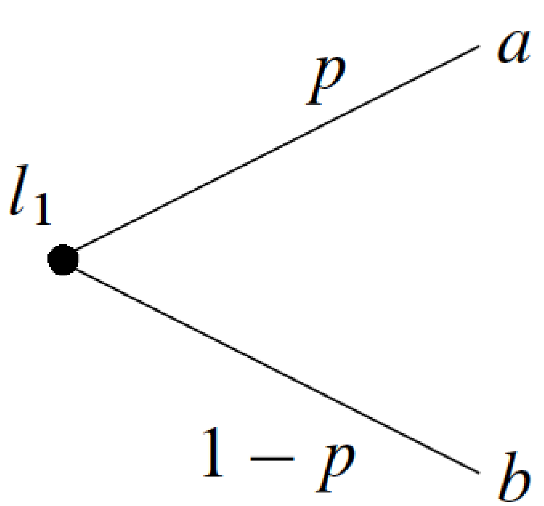

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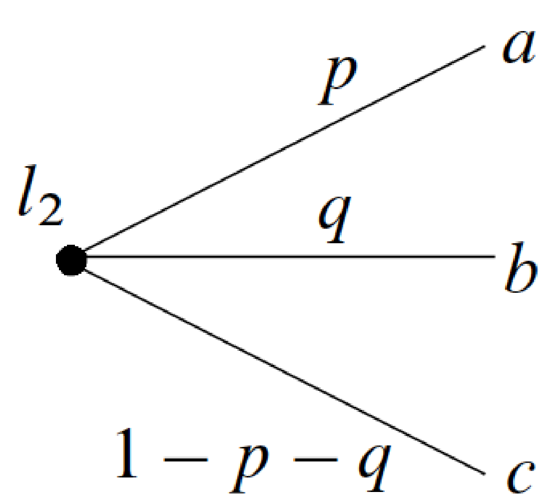

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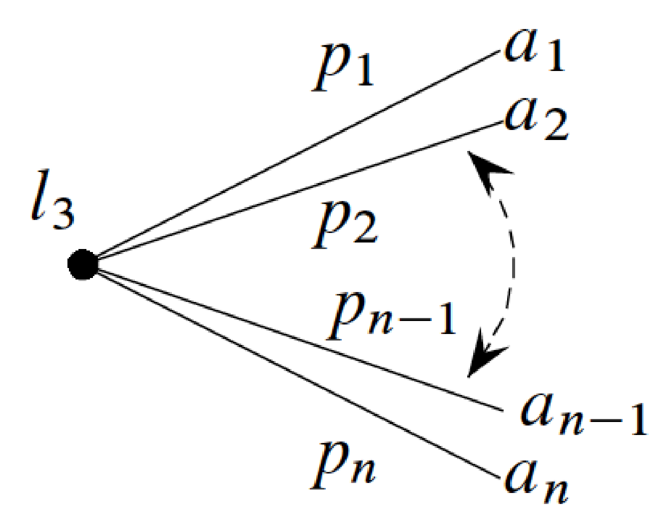

$$E[l_3] = p_1 a_1 + \ldots + p_n a_n$$

As the number of times the lottery goes to infinity, the average payoff converges to the expected value w.p.1

Preferences consistent?

Expected utility principle

- Generalize behavioral properties:
  - Utility function over outcomes
  - Probability distribution over states of nature
- Savage's axioms $\rightarrow$ Expected utility principle
- Consistency of preferences over an appropriate set of lotteries

Notation (summary)

$X=\{x_1, x_2,\ldots, x_n\}$: set of outcomes (payoffs)

$A \subseteq \Omega$: events

Ω: set of states of nature

$L=\{l_1,l_2,\ldots\}$: set of lotteries

A lottery l_i is a function $\pi: \Omega \to X$

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lottery 1

ω_1 —— x_1

ω_2 —— x_2

ω_3 —— x_3

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lottery 1			lottery 2		
ω_1	—	x_1	ω_1	—	x_1
ω_2	—	x_2	ω_2	—	x_1
ω_3	—	x_3	ω_3	—	x_1

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A lottery l_i is a function π: $\Omega \to \mathrm{X}$

lottery 1		lottery 2		lottery 3	
ω_1	x_1	ω_1	x_1	ω_1	x_1
ω_2	x_2	ω_2	x_1	ω_2	x_3
ω_3	x_3	ω_3	x_1	ω_3	x_2

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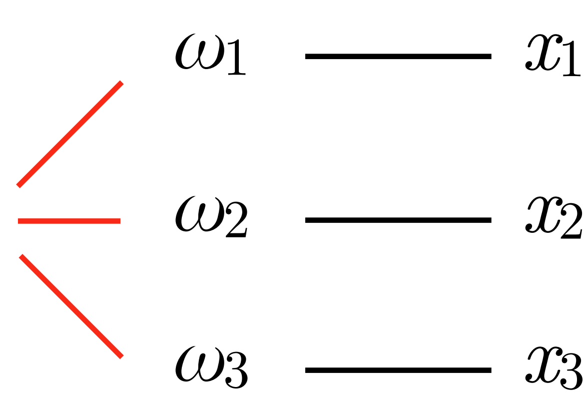
lottery 1	lottery 2	lottery 3
ω_1 — x_1	ω_1 — x_1	ω_1 — x_1
ω_2 — x_2	ω_2 — x_1	ω_2 — x_3
ω_3 — x_3	ω_3 — x_1	ω_3 — x_2

A lottery π associates with each state $\omega \in \Omega$ a payoff $\pi(\omega) \in X$

Lottery

- Individual choose among lotteries
- Do not know state of nature
- Does not include a probability distribution over states of nature $p(\omega)$

lottery π



$$E_\pi[u;p] = \sum_{\omega \in \Omega} p(\omega) u(\pi(\omega))$$

Axiom 1

Desirability of two lotteries does not depend on payoffs where they agree

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For any π, ρ, π', ρ' $\in \mathrm{L}$, let $\mathrm{A}=\{\omega \in \Omega\text{: } \pi(\omega) \neq \rho(\omega)\}$. Suppose we also have $\mathrm{A}=\{\omega \in \Omega\text{: } \pi'(\omega) \neq \rho'(\omega)\}$ and that $\pi(\omega)=\pi'(\omega)$ and $\rho(\omega)=\rho'(\omega)$ for all $\omega \in \mathrm{A}$.

Then $\pi > \rho \leftrightarrow \pi' > \rho'$

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lottery π	lottery ρ	lottery π'	lottery ρ'
ω_1 — x_1	ω_1 — x_1	ω_1 — x_4	ω_1 — x_4
ω_2 — x_2	ω_2 — x_3	ω_2 — x_2	ω_2 — x_3
ω_3 — x_3	ω_3 — x_2	ω_3 — x_3	ω_3 — x_2
ω_3 — x_4	ω_3 — x_4	ω_3 — x_1	ω_3 — x_1

Axiom 2

A natural relation between lotteries and outcomes holds

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If $\mathrm{A} \subseteq \Omega$ is not null, and for all $x, y \in \mathrm{X}$, π, $\rho \in \mathrm{L}$, $\pi | \mathrm{A} = x$ and $\rho | \mathrm{A} = y$.

Then $\pi \succ_A \rho \leftrightarrow x \succ y$

Axiom 2

A natural relation between lotteries and outcomes holds

If $\mathrm{A} \subseteq \Omega$ is not null, and for all $x, y \in \mathrm{X}$, π, $\rho \in \mathrm{L}$, $\pi | \mathrm{A} = x$ and $\rho | \mathrm{A} = y$.

Then $\pi >_A \rho \leftrightarrow x > y$

lottery π		lottery ρ	
ω_1	x_1	ω_1	x_1
ω_2	x_2	ω_2	x_3
ω_3	x_2	ω_3	x_3
ω_3	x_4	ω_3	x_4

$$\pi >_A \rho \leftrightarrow x_2 >_A x_3$$

Axiom 3

The probability that a state of nature occurs is independent of the outcome it receives when it occurs

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Suppose $x > y$, $x' > y'$, $\pi, \rho, \pi', \rho' \in L$, and $\mathrm{A}, \mathrm{B} \in \Omega$.
Suppose that $\pi{=}x,y|\mathrm{A}$, $\rho{=}x',y'|\mathrm{A}$, $\pi'{=}x,y|\mathrm{B}$, $\rho'{=}x',y'|\mathrm{B}$.

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Then $\pi > \pi' \leftrightarrow \rho > \rho'$

	lottery π	lottery π'	lottery ρ	lottery ρ'	
A	ω_1 —— x_1	ω_1 —— x_2	ω_1 —— x_3	ω_1 —— x_4	
	ω_2 —— x_2	ω_2 —— x_2	ω_2 —— x_4	ω_2 —— x_4	
	ω_3 —— x_2	ω_3 —— x_2	ω_3 —— x_4	ω_3 —— x_4	
	ω_3 —— x_2	ω_3 —— x_1	ω_3 —— x_4	ω_3 —— x_3	B

Axiom 4

If a lottery has a higher payoff than another for any event then the first is preferred to the second
(weak version of first order stochastic dominance)

lottery π		lottery ρ	
ω_1	x_1	ω_1	x_1
ω_2	x_2	ω_2	x_3
ω_3	x_2	ω_3	x_1
ω_3	x_4	ω_3	x_4

$$(x_2 >_A x_3) \wedge (x_2 >_A x_1) \rightarrow \pi >_A \rho$$

Axiom 4

If a lottery has a higher payoff than another for any event then the first is preferred to the second
(weak version of first order stochastic dominance)

For any event A, if $x > \rho(\omega)$ for all $\omega \in \mathrm{A}$, then $\pi|\mathrm{A} = x$ implies $\pi >_A \rho$. Also, for any event A, if $\rho(\omega) > x$ for all $\omega \in \mathrm{A}$, and $\pi|\mathrm{A} = x$, then $\rho >_A \pi$.

lottery π		lottery ρ	
ω_1	x_1	ω_1	x_1
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ω_3	x_4	ω_3	x_4

$$(x_2 >_A x_3) \wedge (x_2 >_A x_1) \rightarrow \pi >_A \rho$$

Axiom 5

Technical property: No payoff is "super-good", no payoff is "super-bad"

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Technical property: No payoff is "super-good", no payoff is "super-bad"

For π, ρ, π', $\rho' \in \mathrm{L}$, with $\pi > \rho$, and for all $x \in \mathrm{X}$, there disjoined subsets $\mathrm{A}_1,\ldots,\mathrm{A}_n$ of Ω such that $\mathrm{U}\mathrm{A}_i = \Omega$ and any A_i:

i. If $\pi'(\omega) = x$ for $\omega \in \mathrm{A}_i$ and $\pi'(\omega) = \pi(\omega)$ for $\omega \notin \mathrm{A}_i$, then $\pi' > \rho$

ii. If $\rho'(\omega) = x$ for $\omega \in \mathrm{A}_i$ and $\rho'(\omega) = \rho(\omega)$ for $\omega \notin \mathrm{A}_i$, then $\pi > \rho'$

Allais Paradox

Allais Paradox

- $X=\{x,y,z\}$, $x=2'500'000$, $y=500'000$, $z=0$
- Alternative 1: $\pi=y$
- Alternative 2: $\pi' = 0.1x + 0.89y + 0.01z$

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- Alternative 1: $\pi = y$
- Alternative 2: $\pi' = 0.1x + 0.89y + 0.01z$
- Alternative 1: $\rho = 0.11y + 0.89z$
- Alternative 2: $\rho' = 0.1x + 0.9z$

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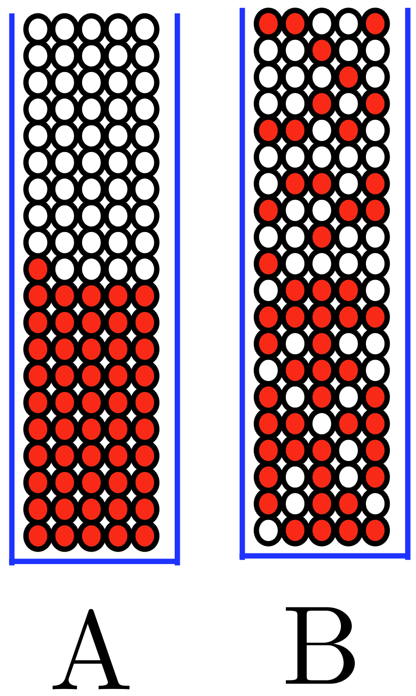
- Alternative 1: $\pi=y$
- Alternative 2: $\pi'= 0.1x + 0.89y + 0.01z$
- Alternative 1: $\rho= 0.11y + 0.89z$
- Alternative 2: $\rho'= 0.1x + 0.9z$
- Consistent?

$\pi > \pi'$

$\rho' > \rho$

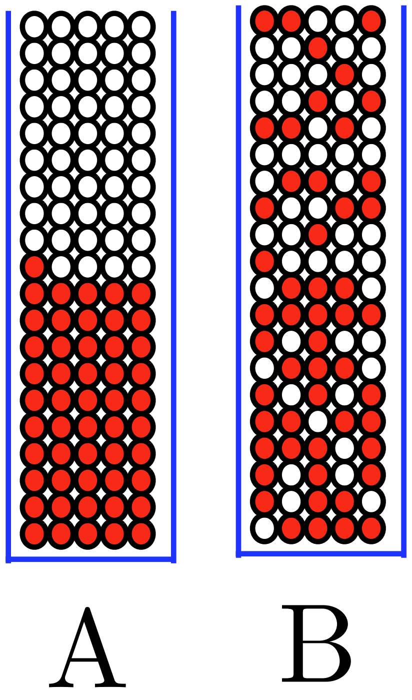
Ellsberg Paradox

- $\mathrm{X}=\{x,y\}$, $x=10$, $y=0$
- Pay 10 for a red ball
- Alternative 1: pick A



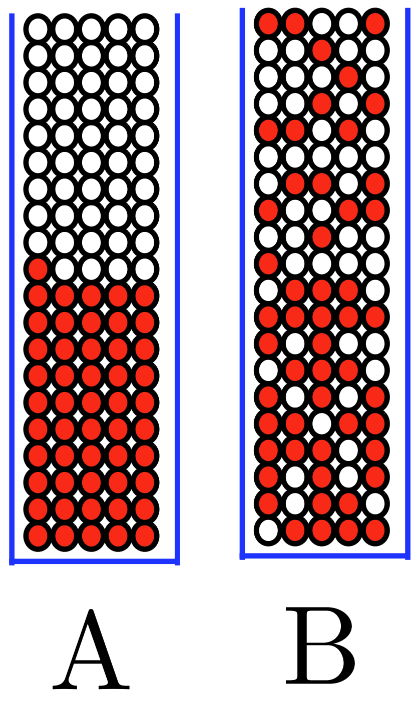
Ellsberg Paradox

- $X = \{x, y\}$, $x = 10$, $y = 0$
- Pay 10 for a red ball
- Alternative 1: pick A
- Alternative 2: pick B



Ellsberg Paradox

- $X=\{x,y\}$, $x=10$, $y=0$
- Pay 10 for a red ball
- Alternative 1: pick A
- Alternative 2: pick B
- Pay 10 for a <u>white</u> ball
- Alternative 1: pick A
- Alternative 2: pick B



Ellsberg Paradox

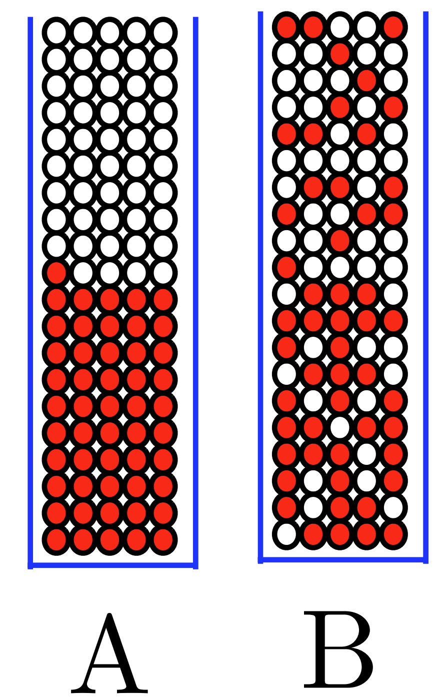
- $X = \{x, y\}$, $x = 10$, $y = 0$
- Pay 10 for a red ball
- Alternative 1: pick A
- Alternative 2: pick B

$A > B$

- Pay 10 for a <u>white</u> ball
- Alternative 1: pick A
- Alternative 2: pick B

$A > B$

- Consistent?



How Basic Are Behavioral Biases? Evidence from Capuchin Monkey Trading Behavior

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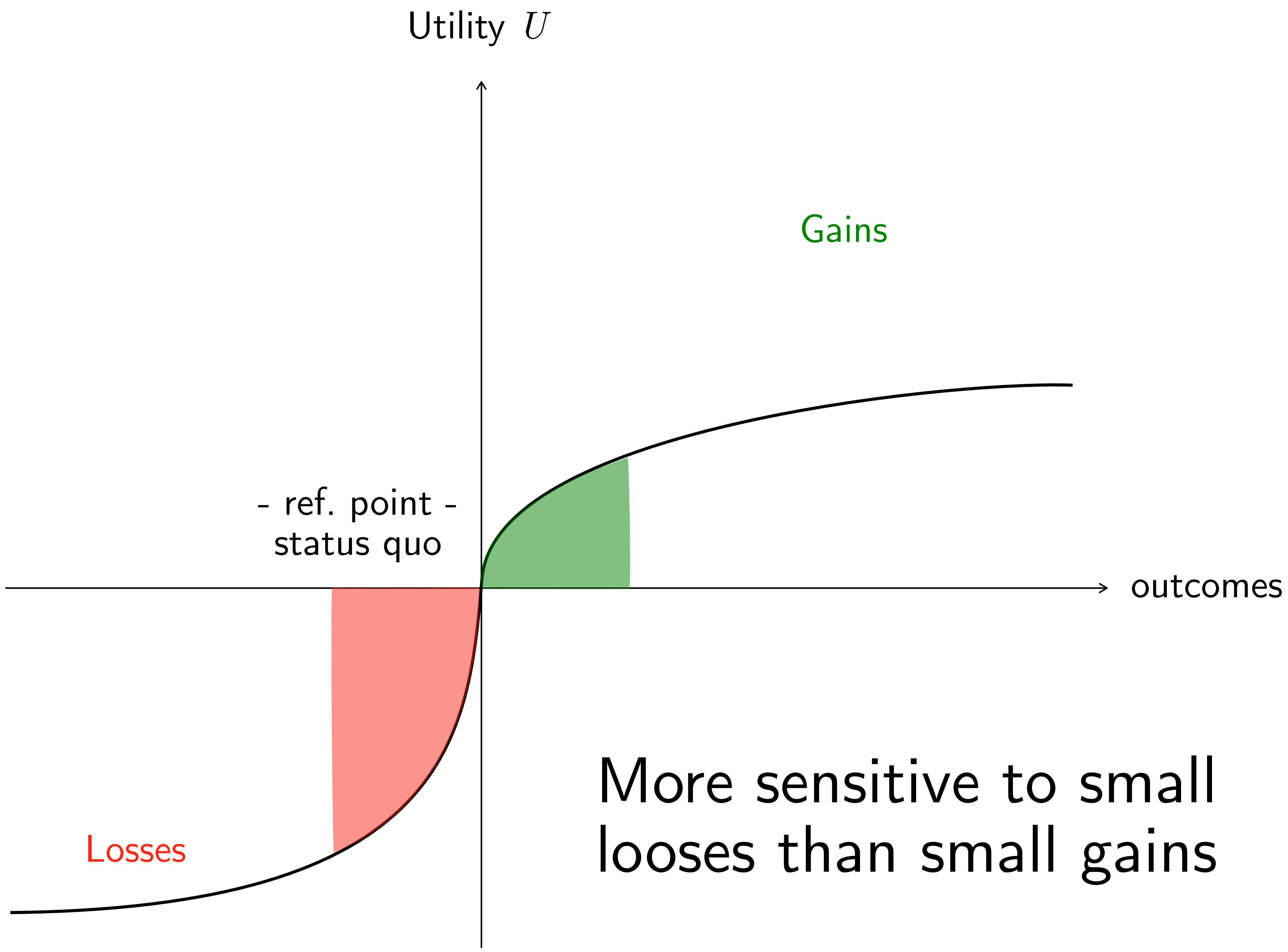
[*Journal of Political Economy,* 2006, vol. 114, no. 3]

Prospect theory
- the shape of the utility function-



More sensitive to small looses than small gains

Next class

- Strategic games
- Representations
  - Normal form
  - Extend form