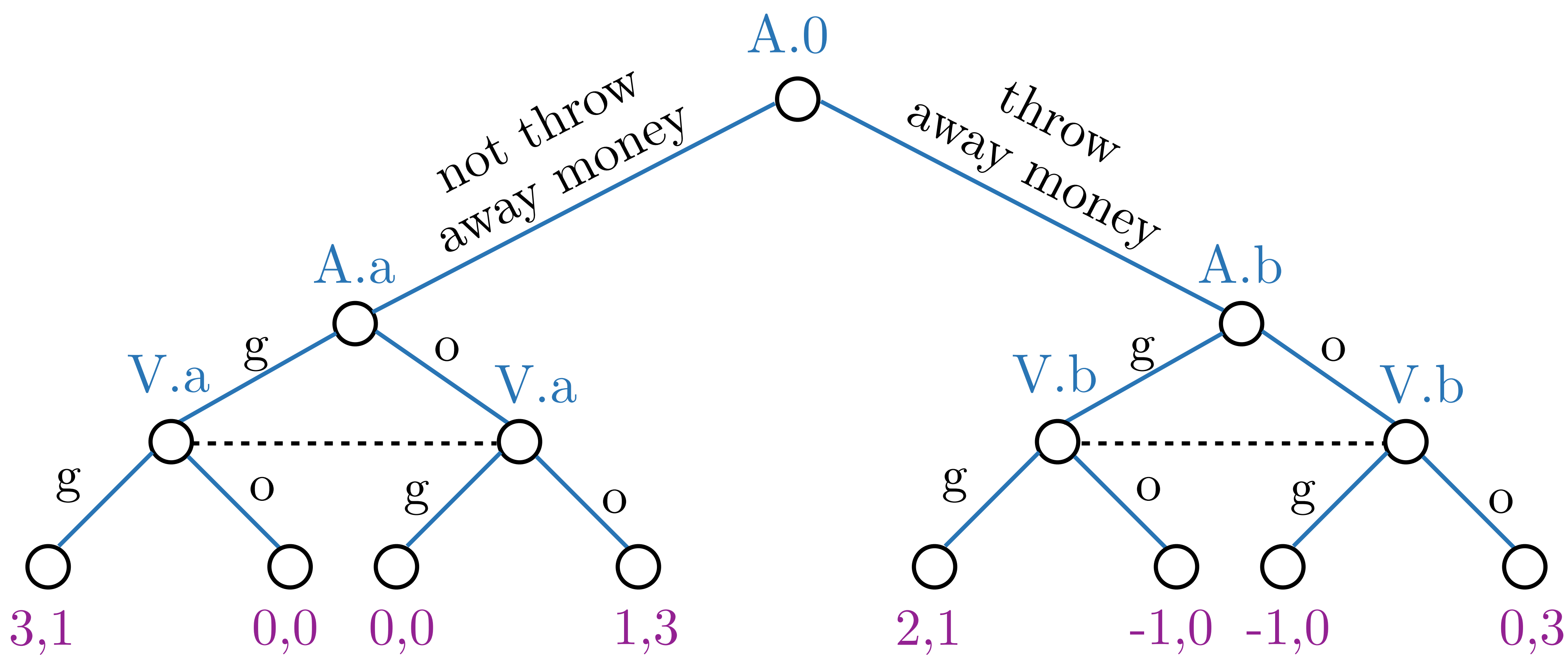
Lecture 10
- common knowledge -

Review

- Strategy elimination in normal form games
- Rationalizable Strategies
- Best response sets

Eliminating weakly dominated strategies



Violetta

Alfredo	gg	go	og	oo
ng	3,1	3,1	0,0	0,0
no	0,0	0,0	1,3	1,3
bg	2,1	-1,0	2,1	-1,0
bo	-1,0	0,3	-1,0	0,3

Throwing away money lead to better outcome for Alfredo?

Eliminating strongly dominated strategies

Player i

Player j		l	c	r
	~~u~~	~~0,2~~	~~3,1~~	~~2,3~~
	m	1,4	2,1	4,1
	d	2,1	4,4	3,2

player i knows that player j is rational

Player j		l	c	~~r~~
	m	1,4	2,1	~~4,1~~
	d	2,1	4,4	~~3,2~~

player i knows that player j knows that player i is rational

Player j		l	c
	~~m~~	~~1,4~~	~~2,1~~
	d	2,1	4,4

player i knows that player j knows that player i knows that player j is rational

Player j		~~l~~	c
	d	~~2,1~~	4,4

$\vdots$

Player j		c
	d	4,4

Rationalizable Strategies

- Rational player: strategy choice $s_i = \mathbf{s}_i(\omega) \in S_i$ maximizes

$$\pi_i(s_i, \phi_i^\omega) = \sum_{s_{-i} \in S_{-i}} \phi_i^\omega(s_{-i}) \pi(s_i, s_{-i})$$

If for every state $\omega \in \Omega$

$$\pi_i(\mathbf{s}_i(\omega), \phi_i^\omega) \geq \pi_i(s_i, \phi_i^\omega)$$

all $s_i \in S_i$

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all $s_i \in S_i$

- A strongly dominated strategy can never be the best response
- Rationalizable: pure strategies that survive the back and forth elimination of strongly dominated strategies

Best response

- For each $x_i \in X_i \subseteq S_i$, player i has a conjecture such that that x_i is a best response to $\phi_{-i} \in \Delta X_{-i}$,

- That is

$$\pi_i(x_i, \phi^{\omega}_{-i}) \geq \pi_i(s_i, \phi^{\omega}_{-i}), \quad s_i \in S_i$$

- Best response set

$$X = \prod_{i=1}^{n} X_i$$

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$$X = \prod_{i=1}^{n} X_i$$

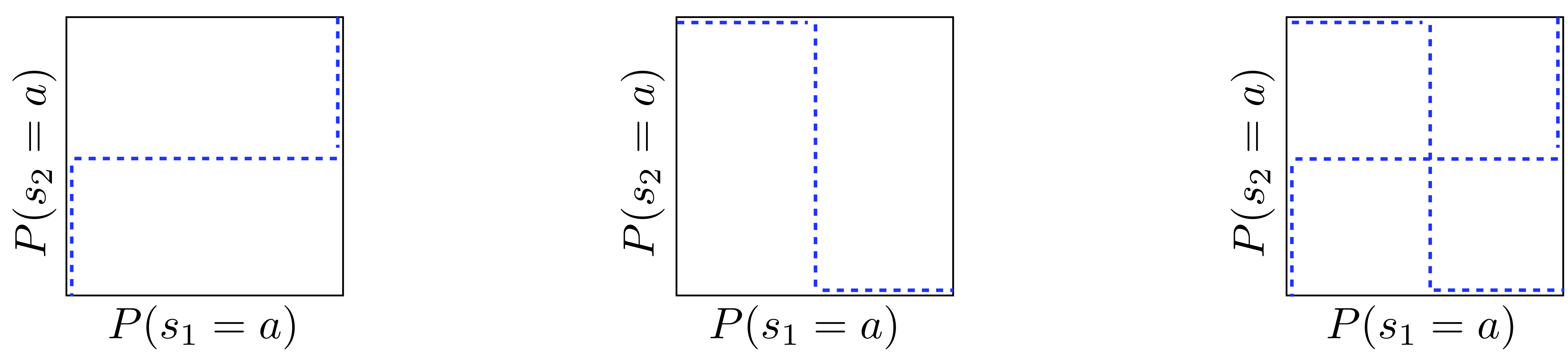
- Rationalizable if a member of the max. best response set

Matching pennies

	a	b
a	1, -1	-1, 1
b	-1, 1	1, -1

- Best response set?
- Completely mixed Nash
  $\rightarrow$ All strategies are rationalizable
- Pure strategies s_i with positive probability in a Nash form a best response set, when the players' conjectures the actual mixed strategy choice of the other players

Best response correspondence



Player 1 | Player 2 | both players

Today

- Common knowledge of rationality
- The beauty contest

The beauty contest

The beauty contest

- $n > 2$ players

The beauty contest

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- Choose simultaneous a number [0, 100]

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The beauty contest

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- Choose simultaneous a number [0, 100]
- Players closest to p x mean win (divided equally if tied)
- $0 \leq p < 1$
- Four rounds by the same group of players

how does a player incorporate the behavior of the other players in conscious reasoning?

Strategies

- Zero-order belief: selects a strategy at random without forming beliefs (picks a number that is salient to him)
- First-order beliefs: thinks that others select a number at random, and he chooses his best response to this belief
- Second-order beliefs: the other player chooses his best response to first-order beliefs
- ...
- r - depth of reasoning

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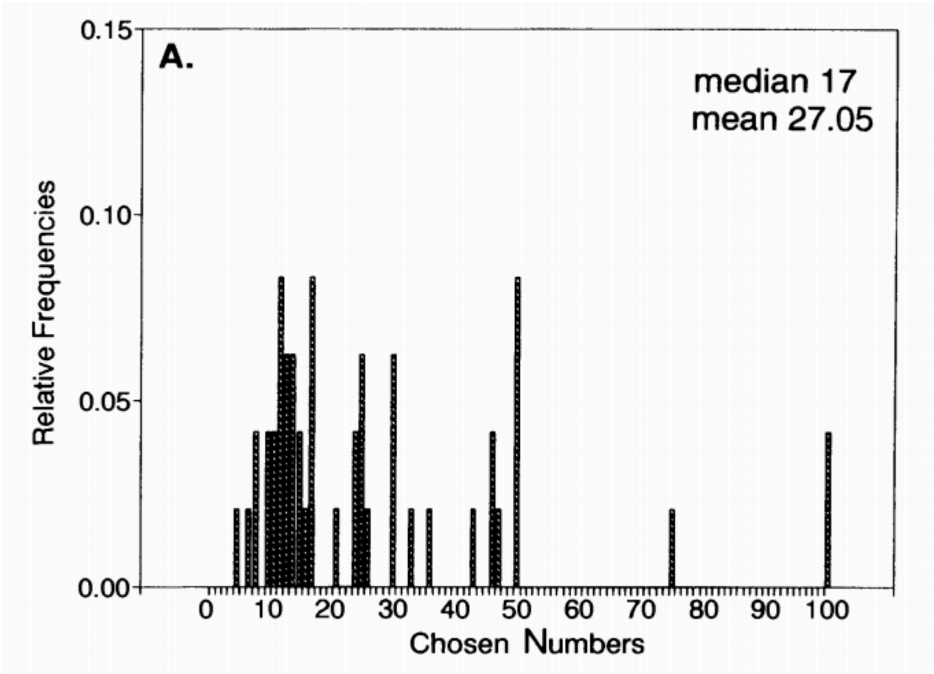
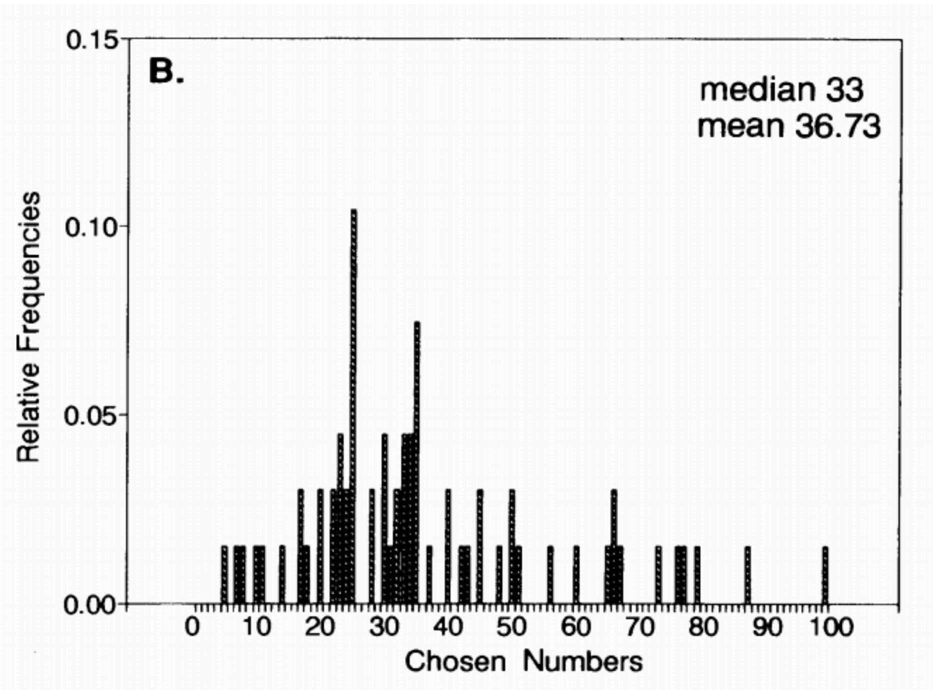
- r - depth of reasoning

how a player's mental process incorporates the behavior of the other players in conscious reasoning

Experimental results



$p = 1/2$

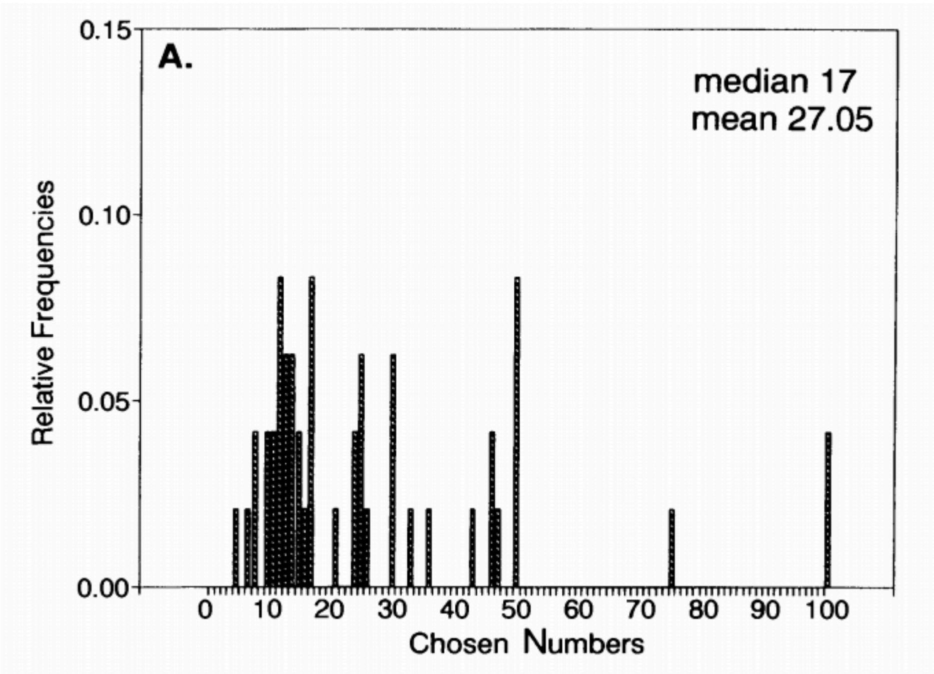


$p = 2/3$

- Depth of reasoning: $2 < r < 3$
- Nash?

Rosemarie Nagel (1995) Unraveling in Guessing Games: An Experimental Study

Experimental results



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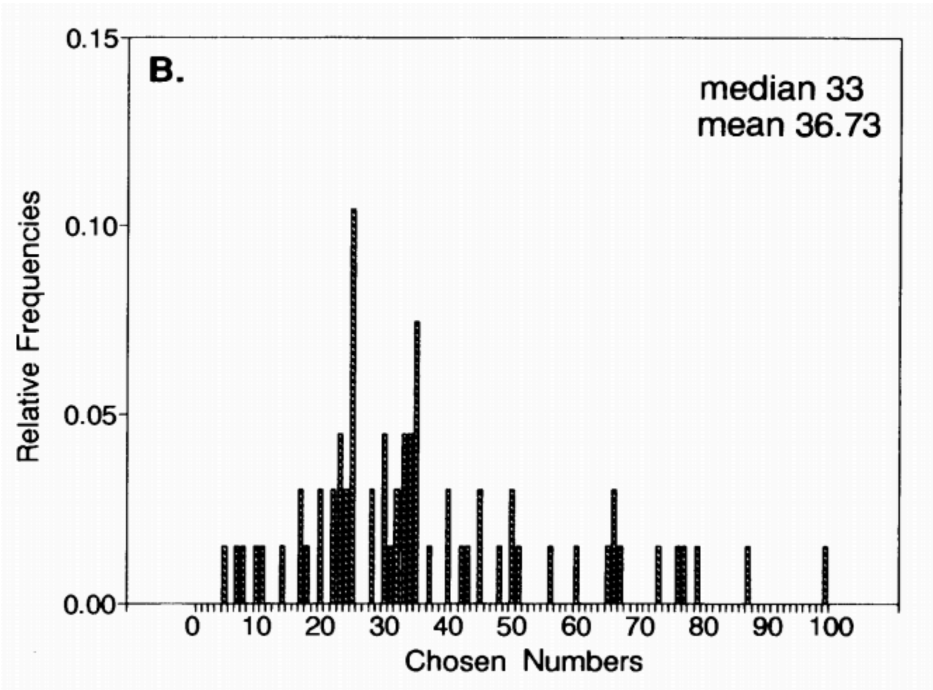

$p = 2/3$

- Depth of reasoning: $2 < r < 3$
- Nash? All players announce zero!
- Only rationalizable strategy!

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