# Stable Emergent Heterogeneous Agent Distributions in Noisy Environments

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***Abstract*— A mathematical model is introduced for the study of the behavior of a spatially distributed group of heterogenous agents which possess noisy assessments of the state of their immediate surroundings. We define general sensing and motion conditions on the agents that guarantee the emergence of a type of "ideal free distribution" (IFD) across the environment, and focus on how individual and environmental characteristics affect this distribution. In particular, we show the impact of the agents' maneuvering and sensing abilities for different classes of environments, and how spatial constraints of the environment affect the rate at which the distribution is achieved. Finally, we apply this model to a cooperative vehicle control problem and present simulation results that show the benefits of an IFD-based distributed decision-making strategy.**

## I. Introduction

The ideal free distribution concept from ecology characterizes how animals optimally distribute themselves across a finite number of habitats. The word "ideal" refers to the assumption that animals have perfect sensing capabilities for simultaneously determining habitat "suitability" (assumed to be a correlate of Darwinian fitness) for all habitats. Moreover, the "ideal" part of the IFD assumption supposes that each animal will move to maximize its fitness. "Free" indicates that animals can move at no cost and instantaneously to any habitat regardless of their current location. If an animal perceives one habitat as more suitable, it moves to this habitat in order to increase its own fitness. This movement will, however, reduce the new habitat's suitability, both to itself and other animals in that habitat. The IFD is an equilibrium distribution where no animal can increase its fitness by unilateral deviation from one habitat to another.

After the IFD notion was introduced in [1]-[2], different models have been developed based on this concept (so called IFD models), each trying to explain how different groups behave as a whole in different environments. In particular, many of these models try to relax the ideal and free assumptions of the IFD by taking into account individual and environmental characteristics, which are essential in understanding the underlying dynamics of the entire group. For instance, in [3] the authors discuss the concept of travel cost and constraints in IFD models (e.g., they consider how the cost of traveling between habitats might diminish the expected benefits of moving to another habitat). Here, the IFD model we introduce extends the one in [4], [5]. Like in [4], [5] it is built on a graph, so that the graph topology defines the interconnections between habitats (nodes) via a set of arcs. By not requiring that every node has an arc to every other node, the graph topology allows us to represent removal of both the ideal and free restrictions to the original IFD model. The author of [6] introduces the concept of "interference" as the direct effect caused by the presence of several competitors in the same habitat. Here, we consider a general class of habitat suitability functions, which allows us to model environments in which interference between individuals may noticeably impact group behavior. Other related studies take into account that animals may differ in "competitive ability," as in [7], [8]. Unlike in [1]-[5], and [6] we consider an approach similar to [7], [8] in that we let every individual have a certain "capacity," which is assumed to be a correlate of its competitive strength, its sensing ability (e.g., an individual may have noisy sensors), its maneuvering ability (e.g., its speed or turn radius), or other individual characteristics that would affect the suitability of the habitat it settles at. We allow individuals to differ in their capacity, have different assessments about habitats, and study how differences in the capacities among individuals affect the optimal distribution.

The main contributions of this paper are as follows. In Section II we develop a discrete agent model that captures individual agents' motion dynamics across the environment. We establish a wide class of agent strategies (i.e., "proximate" decision-making mechanisms) that will lead to an emergent behavior of the group that is a "type of IFD" (which later, for simplicity we will refer as an IFD). By this, we mean one of many possible IFD realizations that are in some sense close to an IFD that is achieved under the original assumptions [1], [2]. Here we must consider a wide class of distributions since the sensing noise and discretization that quantify agent capacity both generally make it impossible to achieve perfect suitability equalization as is demanded by the original IFD concept. In Section III we show how an "invariant set" of spatially distributed discrete individuals can represent the IFD and use Lyapunov stability analysis of this set to illustrate that there is a wide class of resulting agent movement trajectories across nodes that still achieves a desirable distribution. Finally, in Section IV we use the problem of dynamic allocation of vehicles during a cooperative surveillance mission as an engineering application of the model and results.

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## II. A Discrete Agent Model and the Environment

In theoretical ecology, a common approach in modeling is to assume the existence of a large population in the environment. Under such an assumption the total number of individuals in any region or habitat of the environment can be adequately represented by a continuous variable. Such an approach was used in [1], [2] and [4], [5]. Here, we extend the model in [4], [5] to allow for a finite number of discrete agents. As in [4], [5] we assume that individuals (agents) may move and distribute themselves over $N$ available habitats (nodes) and let $H = \{1, \ldots N\}$. Moreover, we define the suitability of node $i \in H$ as $s_i(x_i)$, where $x_i$ represents the state of node $i$. However, we do not require the existence of a large number of individuals in the environment, and we assume instead, that $x_i$ is described with a discrete variable. This allows us to capture individual agent characteristics by taking into account, for example, different agent capacities. Hence, here we assume that $x_i \in \mathbb{R}_+ = [0, \infty)$, represents the total agent capacity at node $i$, which results from multiple discrete agents being present at that node. Let there be a fixed number of agents in the environment. The capacity of each agent stays constant, so that total agent capacity $C = \sum_{i=1}^{N} x_i$ is fixed. Let $\varepsilon_c \in \mathbb{R}_+$ be the minimum agent capacity required to be present at any node $i \in H$ (i.e., either so that all suitability functions are well defined at any state, or as an additional constraint on the environment). We assume that $C > N\varepsilon_c$. Note that the value of $\varepsilon_c$ will depend on the lowest agent capacity of any agent, and the minimum number of agents allowed at any node. In fact, we assume that the total agent capacity in the environment can be partitioned into discrete blocks. Each block represents a particular agent, and its size is assumed to be a correlate of its capacity (competitive capability). We assume that the largest capacity of any agent in the environment is given by $\overline{x} > 0$, and the smallest capacity of any agent is given by $\underline{x}$, so that $\overline{x} \geq \underline{x} > 0$. Moreover, assume the following:

- *Node suitability changes relate to total node agent capacity changes:* We assume that for all $s_i(x_i)$, $i \in H$, there exist constants $\underline{c}_i, \overline{c}_i \in \mathbb{R}$, $\underline{c}_i, \overline{c}_i > 0$, such that

$$-\underline{c}_i \leq \frac{s_i(y_i) - s_i(z_i)}{y_i - z_i} \leq -\overline{c}_i \tag{1}$$

  for any $y_i, z_i \in [\varepsilon_c, C]$, $y_i \neq z_i$. Thus, $s_i(x_i)$ is a strictly monotonically decreasing function in its argument $x_i \in [\varepsilon_c, C]$, so that as the total agent capacity in node $i$ increases, the suitability of the node decreases. Moreover, we assume that $\lim_{x_i \to \infty} s_i(x_i) = 0$ for all $i \in H$.
- *Strictly positive suitability:* We assume that the functions $s_i(x_i) > 0$ for all $i \in H$, and all $x_i \in [\varepsilon_c, C]$.

### A. Environmental Constraints on Agent Sensing and Motion

The interconnection of nodes is described by a bidirectional graph, $(H, A)$, where $A \subset H \times H$ (i.e., a graph where $(i, j) \in A$ implies that $(j, i) \in A$). We assume that for every $i \in H$, there must exist some $j \in H$, $i \neq j$, such that $(i, j) \in A$ and there exists a path between any two nodes, in order to ensure that every node is connected to the graph. If $(i, j) \in A$, this represents that an agent at node $i$ can sense its *neighboring* node $j$ and can move from $i$ to $j$. According to the definition of $(H, A)$, if an agent is at $i$ and can move to $j$ (sense the suitability at $j$), agents at $j$ can also move from $j$ to $i$ (sense the suitability at $i$, respectively). We also assume that if $(i, j) \in A$, agents at node $i$ know the total agent capacity at node $j$, $x_j$, and also $x_i$. However, we do not assume that agents have perfect sensor capabilities to measure its own or the suitability levels of its neighboring nodes. In particular, for agents at node $i$, where $(i, j) \in A$, "sensing node $j$" implies that agents at node $i$ know $s_j(x_j) + w$, where $w$ is "sensing noise" that can change over time randomly, but $-\underline{w} \leq w \leq \overline{w}$ for known constants $\underline{w}, \overline{w} \geq 0$. Let $s_j^i(x_j) = s_j(x_j) + w$ denote the perception (i.e., the noisy measured value) by agents at node $i$ of the suitability level of node $j$ with total agent capacity $x_j$. In some cases one might want to assume that $w$ depends on $x_i$. For instance, $s_j^i(x_j) = s_j(x_j) + w(x_i)$ with $\underline{w} = \overline{w}$, and $|w(x_i')| > |w(x_i'')| \geq 0$ for $x_i'' > x_i'$ represents sensing conditions where a larger agent capacity at node $i$ results in a better suitability perception of its neighboring node $j$ (e.g., due to better sensing capacities of the individual agents, agreement strategies among different agents at the same node that improve their individual sensing abilities, or averaging strategies which compensate for the error present in individual suitability assessments). Other sensing conditions may require that $s_j^i(x_j) = s_j(x_j) + w^{ij}$, where $w^{ij}$ is the sensing noise present when agents at node $i$ measure the suitability level of node $j$, in order to represent that different habitats may be measured with different accuracy. Here, we simply assume that if $w(k)$ is the sensing noise present in an agent's perception at time $k$, then it may be that $w(k_1) \neq w(k_2)$ for $k_1 \neq k_2$, which produces a general framework to represent that the sensing capabilities of the agents may change over time (e.g., as agents discover their surroundings, their ability to assess the suitability levels of neighboring nodes may change).

Note that an agent's perception about the suitability level of a neighboring node may differ from its actual value by at most $\max\{\underline{w}, \overline{w}\}$. Also, note that given a node $\ell \in H$, and two neighboring nodes $i, j$ such that $(\ell, i) \in A$ and $(\ell, j) \in A$ with $s_i(x_i) > s_j(x_j)$, if $s_i(x_i) - s_j(x_j) > 2\max\{\underline{w}, \overline{w}\}$, then the measured values of the suitability levels of nodes $i$ and $j$ by agents at node $\ell$ are such that, $s_i^\ell(x_i) > s_j^\ell(x_j)$, regardless of the sensing noise $w$ present during the measurements. In other words, if $s_i(x_i) - s_j(x_j) > 2\max\{\underline{w}, \overline{w}\}$, then the two sets of all possible measured values of the suitability levels of the corresponding nodes $i$ and $j$, given $s_i(x_i)$ and $s_j(x_j)$, do not overlap. Conversely, note that these sets may only overlap if $0 < s_i(x_i) - s_j(x_j) \leq 2\max\{\underline{w}, \overline{w}\}$. Moreover, if $(j, i) \in A$, then $|s_i^j(x_i) - s_i(x_i)| \leq \max\{\underline{w}, \overline{w}\}$, and therefore $|s_i^j(x_i) - s_j(x_j)| \leq 3\max\{\underline{w}, \overline{w}\}$. Finally, since $|s_j^j(x_j) - s_j(x_j)| \leq \max\{\underline{w}, \overline{w}\}$, we obtain that $|s_i^j(x_i) - s_j^j(x_j)| \leq 4\max\{\underline{w}, \overline{w}\}$, regardless of the noise $w$ present during the measurement. Let us define $W = 4\max\{\underline{w}, \overline{w}\}$ as the maximum difference between the measured suitability value of a neighboring node and the perception of the suitability level of the node where the sensing agents are located, given that the actual suitability levels of both nodes $i$ and $j$ are close enough (i.e., they do not differ by more than $2\max\{\underline{w}, \overline{w}\}$).

We use the distributed discrete event system modeling methodology from [9]. Let $\mathbb{R}_{\varepsilon_c} = [\varepsilon_c, \infty)$ and $\mathcal{X} = \left\{x \in \mathbb{R}^N_{\varepsilon_c} : \sum_{i=1}^N x_i = C\right\} \subset \mathbb{R}^N_+$ be the simplex over which the $x_i$ dynamics evolve. Let $x(k) = [x_1(k), x_2(k), \ldots, x_N(k)]^\top \in \mathcal{X}$ be the state vector, where $x_i(k)$ represents the total agent capacity at node $i$ at time index $k \geq 0$. Constraints on our model below will ensure that $x(k) \in \mathcal{X}$ for all $k \geq 0$. Let $I(x) = \{i \in H : x_i > \varepsilon_c, \ x \in \mathcal{X}\}$ represent the set of nodes at state $x$, such that each node $i \in I(x)$ is occupied by a certain number of agents which results in the total agent capacity at node $i$ exceeding the value of $\varepsilon_c$. Similarly, let $U(x) = H - I(x)$ represent the set of nodes at state $x$ whose total agent capacity equals the minimum agent capacity $\varepsilon_c$. The size of the set $I(x)$ is denoted by $N_I$. Let $M = \max_i \{s_i(x_i) - s_i(x_i + \overline{x}) : \text{ for all } x_i \in [\varepsilon_c, C]\}$ for all $i \in H$. In other words, $M$ is the maximum change in suitability that could occur by having an agent of maximum capacity leave any node. Figure 1 shows an example of a system with $N = 3$ nodes and perfect sensing capabilities so that $\underline{w} = \overline{w} = 0$. Note that a horizontal band of width $M > 0$ crossing at least one $s_i$ curve represents an IFD state for some total agent capacity in the environment $C$.

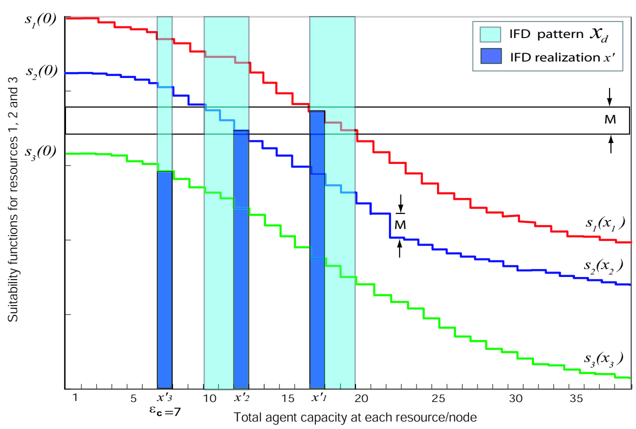


Fig. 1. Suitability functions $s_i(x_i)$ for three fully connected nodes with $\overline{x} = \underline{x} = 1$, $\underline{w} = \overline{w} = 0$, $\varepsilon_c = 7$, and $C = 36$. Under perfect sensing conditions the IFD distribution is reached when all agents are distributed in such a way that at state $x$ neighboring nodes $i$ such that $i \in I(x)$ have suitability levels that do not differ by more than $M$. After the IFD is reached there is no movement of agents between nodes. For the example shown in the plot, while agents distribute themselves over nodes 1 and 2, node 3 remains with the minimum agent capacity $\varepsilon_c$ at the desired distribution. Node $i = 3$ is called a *truncated* node. The suitability level $s_3(\varepsilon_c)$ is too low to be chosen by any agent at other nodes. Note also that there may exist different distributions of the total agent capacity that correspond to neighboring suitability levels of nodes $i \in I(x)$ differing by at most $M$. Each such distribution is called an *IFD realization*. The light-colored vertical bands represent all possible distributions of agent capacity for which the IFD pattern is achieved. We denote the set of all IFD realizations by $\mathcal{X}_d$ and will describe it mathematically in Section III. The dark-colored vertical bars illustrates a particular distribution $x' = [7, 12, 17]^\top$, and its resultant suitability levels that satisfied the IFD pattern (e.g., note that $x' = [7, 11, 18]^\top$ and $x' = [7, 10, 19]^\top$ would also result in suitability levels that satisfy the IFD pattern).

For a general graph topology, the best we can generally hope to do with only local information and a distributed decision-making strategy under perfect sensing capabilities is to distribute agent capacities in such a way that the suitability levels between any two connected nodes remain within $M$. In particular, we can guarantee that $|s_i(x_i) - s_j(x_j)| \leq M$ for all $(i,j) \in A$ such that $i, j \in I(x)$ at the desired distribution. Note that the value of $M$ depends on the particular shape of all the suitability functions (i.e., the suitability function of any node is bounded by Equation (1)), the total agent capacity in the environment $C$, and the largest capacity of any agent $\overline{x}$. In particular, note that since Equation (1) applies for all $i \in H$ and any $y_i, z_i \in [\varepsilon_c, C]$, if we let $y_i = x_i$ and $z_i = x_i + \overline{x}$, we can bound $M$ by $\overline{x}\min_i\{\overline{c}_i\} \leq M \leq \overline{x}\max_i\{\underline{c}_i\}$. Similarly, $m = \min_i\{s_i(x_i) - s_i(x_i + \underline{x}) : \text{ for all } x_i \in [\varepsilon_c, C]\}$ for all $i \in H$. Equation (1) guarantees that $M, m > 0$.

### B. Agent Sensing, Coordination, and Motion Requirements

Let $\mathcal{E}$ be a set of events and let $e^{i,p(i)}_{\alpha(i,k)}$ represent the event that one or more agents move from node $i \in H$ to neighboring nodes $\ell \in p(i)$ at time $k$, where $p(i) = \{j : (i,j) \in A\}$. Note that movement of agents from node $i$ to neighboring nodes decreases $x_i$ since node $i$ reduces its total agent capacity and consequently increases $s_i(x_i)$. Let $\alpha_\ell(i,k)$ denote the total agent capacity of the agents that move from node $i \in H$ to node $\ell \in p(i)$ at time $k$. Let the list $\alpha(i,k) = (\alpha_j(i,k), \alpha_{j'}(i,k), \ldots, \alpha_{j''}(i,k))$ such that $j < j' < \cdots < j''$ and $j, j', \ldots, j'' \in p(i)$ and $\alpha_j \geq 0$ for all $j \in p(i)$ represent the total agent capacity of the agents that move to all neighboring nodes of node $i$; the size of the list $\alpha(i,k)$ is $|p(i)|$ and remains constant for all time $k \geq 0$ for all $i \in H$, since the topology of the graph $(H, A)$ is assumed to be time invariant (i.e., $\alpha(i,k) \in \mathbb{R}^{|p(i)|}_C$ for all $k$, where $\mathbb{R}_C = [0, C]$). Let $\{e^{i,p(i)}_{\alpha(i,k)}\}$ represent the set of *all* possible combinations of how agents can move from node $i$ to its neighboring nodes for all $k$. Let the set of events be described by $\mathcal{E} = \mathcal{P}\left(\left\{e^{i,p(i)}_{\alpha(i,k)}\right\}\right) - \{\emptyset\}$ ($\mathcal{P}(\cdot)$ denotes the power set). Notice that each event $e(k) \in \mathcal{E}$ is defined as a *set*, with each element of $e(k)$ representing the transition of possibly multiple agents among neighboring nodes in the graph. Multiple elements in $e(k)$ represent the simultaneous movements of agents, i.e., migrations out of multiple nodes.

An event $e(k)$ may only occur if it is in the set defined by an "enable function," $g : \mathcal{X} \to \mathcal{P}(\mathcal{E}) - \{\emptyset\}$. State transitions are defined by the operators $f_e : \mathcal{X} \to \mathcal{X}$, where $e \in \mathcal{E}$. We now specify $g$ and $f_e$ for $e(k) \in g(x(k))$, which define the agents' sensing and motion:

- If for a node $i \in H$, $s^i_j(x_j) - s^i_i(x_i) \leq M$ for all $(i,j) \in A$, then $e^{i,p(i)}_{\alpha(i,k)} \in e(k)$ such that $\alpha(i,k) = (0, \ldots, 0)$ is the only enabled event. Hence, agents at the most suitable node that they know of do not move.
- If for node $i \in H$, $s^i_j(x_j) - s^i_i(x_i) > M$ for some $j$ such that $(i,j) \in A$, then the only $e^{i,p(i)}_{\alpha(i,k)} \in e(k)$, are

ones with $\alpha(i,k) = (\alpha_j(i,k) : j \in p(i))$, such that:

$$(i) \qquad x_i(k) - \sum_{\ell \in p(i)} \alpha_\ell(i,k) \geq \varepsilon_c$$

$$(ii) \qquad s_i^i\left(x_i(k) - \sum_{\ell \in p(i)} \alpha_\ell(i,k)\right) < \max_j\{s_j^i(x_j(k)) : j \in p(i)\} - W$$

$$(iii) \qquad \text{If } \alpha_j(i,k) > 0 \text{ for some } j \in p(i), \text{ then } \alpha_{j^*}(i,k) \geq \underline{\alpha} \text{ for some } j^* \in \{j : s_j^i(x_j(k)) \geq s_\ell^i(x_\ell(k)) \text{ for all } \ell \in p(i)\}$$

$$(iv) \qquad \alpha_j(i,k) = 0 \text{ for any } j \in p(i) \text{ such that } s_i^i(x_i(k)) > s_j^i(x_j(k)) \text{ and } x_j(k) = \varepsilon_c$$

Condition $(i)$ guarantees that at any node there is at least $\varepsilon_c$ agent capacity. It is required so that conditions $(ii)$ and $(iii)$ are well defined at all times. To interpret conditions $(ii)-(iv)$ it is useful to note that reducing (increasing) the total agent capacity at a node always increases (decreases, respectively) the suitability at that node. The three conditions constrain how agents can move based on their capacities and in terms of node suitabilities. Note that agents may also move from higher suitability nodes to lower suitability nodes as long as all conditions are satisfied. Without condition $(ii)$, there could be a sustained migration oscillation between nodes. Condition $(iii)$ implies that at least one agent must move to the neighboring node perceived with the highest suitability. Without condition $(iii)$ some high suitability node could be ignored by the agents and the IFD distribution might not be achievable. Condition $(ii)$ together with condition $(iii)$ guarantees that the highest suitability node is strictly monotonically decreasing over time. Finally, without condition $(iv)$ some agents would still be free to move to nodes with lower suitability levels, and the desired distribution would not be maintained.

- If $e(k) \in g(x(k))$, $e^{i,p(i)}_{\alpha(i,k)} \in e(k)$, then $x(k+1) = f_{e(k)}(x(k))$, where $x_i(k+1)$ equals $x_i(k)$ plus

$$\sum_{\{j: i \in p(j), e^{j,p(j)}_{\alpha(j,k)} \in e(k)\}} \alpha_i(j,k) - \sum_{\{j: j \in p(i), e^{i,p(i)}_{\alpha(i,k)} \in e(k)\}} \alpha_j(i,k)$$

Note that if $x(0) \in \mathcal{X}$, $x(k) \in \mathcal{X}$, $k \geq 0$.

Let $\mathcal{E}^{\mathbb{N}}$ denote the set of all infinite sequences of events in $\mathcal{E}$. Let $E_v \subset \mathcal{E}^{\mathbb{N}}$ be the set of valid event trajectories for the model (i.e., ones that are physically possible). Event $e(k) \in g(x(k))$ is composed of a set of what we will call "partial events." Define a *partial event of type* $i$ to represent the movement of $\alpha(i,k)$ agents from node $i \in H$ to its neighbors $p(i)$ so that conditions $(i)-(iv)$ are satisfied at time $k$. A partial event of type $i$ will be denoted by $e^{i,p(i)}$ and the occurrence of $e^{i,p(i)}$ indicates that *some* agents located at node $i \in H$ move to other nodes. Partial events must occur according to the "allowed" event trajectories. The allowed event trajectories define the degree of asynchronicity of the model at the node level. We define two possibilities for the allowed event trajectories:

First, for allowed event trajectories $E_i \subset E_v$, assume that each type of partial event occurs infinitely often on each event trajectory $E \in E_i$. The assumption is met if at each node all agents do not ever stop trying to move (e.g., if each agent persistently tries to move to neighboring nodes). This corresponds to assuming "total asynchronism" [10].

Second, for allowed event trajectories $E_B \subset E_v$, assume that there exists $B > 0$, such that for every event trajectory $E \in E_B$, in every substring $e(k'), \ldots, e(k' + (B-1))$ of $E$ there is the occurrence of every type of partial event (i.e., for every $i \in H$, the partial event $e^{i,p(i)} \in e(k)$, for some $k$, $k' \leq k \leq k' + B - 1$). This corresponds to assuming "partial asynchronism" [10].

## III. Emergent Agent Distribution

The set

$$\mathcal{X}_d = \{\, x \in \mathcal{X} : \text{ for all } i \in H, \textit{ either } |s_i(x_i) - s_j(x_j)| \leq M + W \text{ for all } j \in p(i) \text{ such that } x_j \neq \varepsilon_c \text{ and } s_i(x_i) > s_j(x_j) \text{ for all } j \in p(i) \text{ such that } x_j = \varepsilon_c, \textit{ or } x_i = \varepsilon_c \} \qquad (2)$$

is an invariant set that represents all possible distributions of the total agent capacity $C$ at the IFD since for $x \in \mathcal{X}_d$, $|s_i(x_i) - s_j(x_j)| \leq M + W$ for all $i, j \in I(x)$ such that $(i,j) \in A$, and $s_i(x_i) = s_i(\varepsilon_c)$ for all $i \in U(x)$. It can be shown that according to the definition of the enable function $g$ there is no agent movement between nodes, so that $\alpha(i,k) = (0, \ldots, 0)$ for all $i \in H$ when $x(k) \in \mathcal{X}_d$. Moreover, note that there exist many different agent distributions that belong to $\mathcal{X}_d$. Any agent distribution such that the distribution of the total agent capacities $x \in \mathcal{X}_d$ is an IFD realization. Note that according to the definition of $\mathcal{X}_d$ it is possible for unconnected nodes (i.e., ones such that $(i,j) \notin A$) in the set $I(x)$ to have suitabilities that differ by more than $M$ when the distribution is achieved. This could happen if two nodes $i, j$ such that $i, j \in I(x)$ with high suitability levels when $x \in \mathcal{X}_d$ are separated by a node with minimum agent capacity (e.g., in an environment represented by a line topology of the graph $(H, A)$). However, any two nodes that are linked according to the graph $(H, A)$ (i.e., ones such that $(i,j) \in A$) *and* belong to the set $I(x)$ must have suitability levels that differ at most by $M+W$ at the desired distribution. Hence, depending on the graph's connectivity, there could be isolated "patches" of nodes where only nodes belonging to the same patch have suitability levels that differ by at most $M + W$ (i.e., forming an environment of different patches). Moreover, note that the formation of patches depends on the total agent capacity in the environment, the initial distribution $x(0)$, and random agent migration between nodes.

**Theorem 1 (Stability for a fully connected environment, any total agent capacity):** Given a fully connected graph $(H, A)$, $\varepsilon_c > 0$, any population size with total agent capacity $C$, and agent motion conditions $(i)-(iv)$, the invariant set $\mathcal{X}_d$ is asymptotically stable in the large with respect to $E_i$ and exponentially stable in the large with respect to $E_B$.

Due to space constraints we do not include any proofs here. For detailed information about the proofs of any of the theorems the reader should contact the authors.

Note that asymptotic/exponential stability *in the large* implies that for any initial distribution of agent capacity, the invariant set will be achieved. This result provides general sufficient conditions on when a distribution satisfying the IFD pattern is achieved. However, the size of $\mathcal{X}_d$ is not necessarily one, since there are many possible IFD realizations that may be achieved. Theorem 1 guarantees that under the above stated sensing and motion conditions one of them will be reached. Moreover, our analysis considers all environments which can be modeled by a wide class of suitability functions. It includes functions which have been found to be useful in biology, like the one originally used to introduce the IFD concept in [1], and the one in [8] which introduced the interference model, among others.

Note also that Theorem 1 requires $\varepsilon_c > 0$ because if $\varepsilon_c = 0$ at a truncated node $i$, then $s_i(x_i)$ equals infinity for certain suitability functions (e.g., $s_i(x_i) = \frac{a_i}{x_i}$). The proof of Theorem 1 considers the dynamic emergence of different patches when the environment is modeled by a fully connected topology. Patches emerge as agents distribute themselves over the nodes, and the total agent capacity is small enough.

**Theorem 2 (Stability for a not fully connected environment, but sufficient total agent capacity):** Given any $(H, A)$, $\varepsilon_c \geq 0$, and agent motion conditions $(i) - (iv)$, there exists a constant $C > N\varepsilon_c$ such that if the total agent capacity in the environment is at least $C$, then the invariant set $\mathcal{X}_d$ is asymptotically stable in the large with respect to $E_i$ and exponentially stable in the large with respect to $E_B$.

Theorem 2 considers a general interconnection topology, which allows us to consider less restrictive agent sensing and motion abilities. For this case we show that for a large enough total agent capacity $C$ there are no isolated patches in the environment at the desired distribution. Theorem 2 is an extension of the load balancing [10] theorems in [9], [11] to the case when the "discrete virtual load" is a nonlinear function of the state.

## IV. Application: Cooperative Vehicle Control

Suppose we wish to design a multi-vehicle guidance strategy to enable a group of vehicles to perform surveillance of some region where the goal is to make the proportion of vehicles visiting a set of predefined areas match the relative importance of monitoring each area. This vehicle distribution goal must be achieved in spite of vehicle sensing, communication, and motion constraints (the combination of which requires a decentralized vehicle guidance strategy with each vehicle making independent decisions). Assume that the $i^{th}$ vehicle obeys a Dubin's model with (constant) velocity $v$ and minimum turn radius $T$ (i.e., vehicles will either travel on the minimum turning radius or on straight lines). Assume also that the region under surveillance can be divided into $N$ equal-size square $\ell \times \ell$ areas. These areas are the nodes $i \in H$. The connectedness of the areas is modeled by the topology of the graph $(H, A)$. We assume that new targets continually pop-up at points in the surveillance region according to some stochastic process. We let $R_i$ characterize the (average) rate of appearance of pop-up targets in area $i$, and assume it is constant but unknown to the vehicles. We assume that pop-up target locations in area $i$ are known only to vehicles currently in $i$ and that they stay exposed until they are visited by some vehicle. When a vehicle starts approaching a target, the target is considered to be "attended," and a vehicle may visit a new target only after the target being approached has been reached. Once the target is reached, the vehicle may perform various tasks and it is then ignored for the rest of the mission.

The suitability level of an area is defined as the (average) *rate of appearance of unattended targets* (i.e., targets which have appeared but are not being or have not been attended by any vehicle). Figure 2 shows two classes of suitability functions for different intra-area vehicle coordination strategies and target pop-up rates $R_i$. The left plot assumes that vehicles located in the *same* area coordinate in order to decide which targets within that area to attend (i.e., after a target is reached, a vehicle approaches the closest target that is not being approached by any other vehicle). The right plot assumes that vehicles located over the same region do not coordinate and they randomly approach any target located within the area they are monitoring. Here, since our focus is on the relative proportioning of area monitoring and not intra-area coordination, we use the no intra-area coordination approach in the remainder of the paper (conceptually similar results to those below are obtained for specific intra-area coordination).

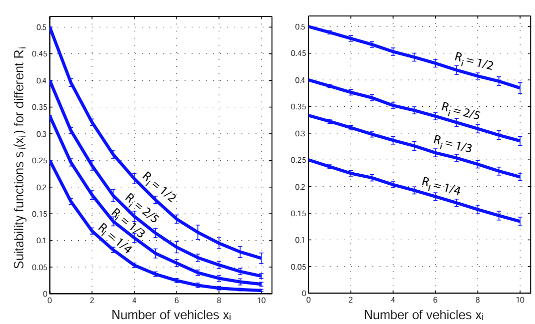


Fig. 2. Suitability functions $s_i(x_i)$ for an area with $\ell = 2.5$km and vehicles at speed $v = 15$m/s, $T = 100$m with (left) and without (right) intra-area coordination strategies to decide which targets within the area to approach. Each data point represents 60 simulation runs with varying target pop-up locations. The error bars are sample standard deviations from the mean.

Assume that the vehicles only have noisy perceptions about the suitability levels of the area they are monitoring and its neighboring areas. To define the perception by any vehicle about the suitability level of an area, we use a system identification approach to determine a parameterized model of the *expected* suitability function of that area, $\hat{s}_i(x_i)$. In particular, under the above assumptions and according to Figure 2, for a fixed $T$ the expected suitability functions are of the form

$$\hat{s}_i(x_i) = \hat{R}_i - \hat{r}(v, \ell)x_i \quad (3)$$

for all $i \in H$, where $\hat{R}_i$ is the expected target pop-up rate for area $i$ (targets/s), and $\hat{r}(v, \ell)$ is the expected rate of targets being attended by each vehicle moving at speed $v$ in an area of size $\ell \times \ell$ (targets/s/vehicle). A vehicle's perception about the suitability level of an area will depend on how the different parameters in Equation (3) are affected by its limited sensing and maneuvering capabilities.

While maneuvering constraints on the vehicles (i.e., an increasing minimum turn radius) may diminish the expected rate $\hat{r}(v, \ell)$ for all vehicles in an area, the expected suitability function shape stays the same as in Equation (3). Furthermore, note that in many applications, knowing the value of $\hat{R}_i$ in Equation (3) usually requires that vehicles estimate the number of targets that have appeared in that area in a time window divided by the length of that window. Here we assume that vehicles have good sensing capabilities and use a large enough window in estimating the rate of appearance of targets (e.g., so that vehicles monitoring area $i$ can ultimately obtain $\hat{R}_i$ and $\hat{R}_j$ for all $j \in p(i)$ within $10\%$ of $R_i$ and $R_j$, respectively).

We define the perception by a vehicle located over area $i$ about the suitability level of a neighboring area $j$ as $s^i_j(x_j) = \hat{s}_j(x_j)$ and this will be used in the movement rules defined in Section II-B. As the mission progresses, vehicles decide to move from one area to another only if the proposed conditions $(i)-(iv)$ are satisfied. Figure 3 shows two typical different IFD realizations for $20$ vehicles in a region divided into four areas, and where a line topology is used. While the plots illustrate that good vehicle surveillance distributions are achieved, different IFD realizations can emerge due to the discrete nature of vehicle capabilities (compare left and right plots).

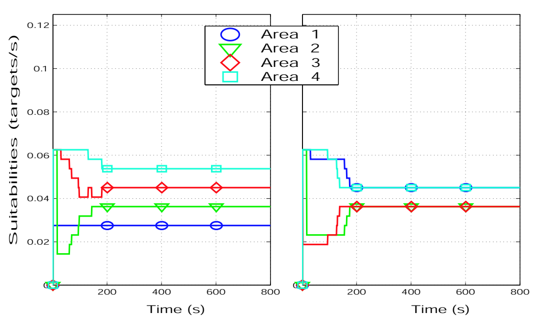


Fig. 3. Two possible IFD realizations for vehicles deployed in an environment divided into four areas connected by a line topology.

Next, using ideas from [10] we define two cooperative sensing strategies to try to reduce the effects of the perception noise $w$ on the mission performance. In particular, we assume that every vehicle that is able to measure the suitability level of an area, will cooperate with other vehicles by sharing with them its own perception about that area. We first implement a synchronous averaging strategy, where at any time $k$ all vehicles may exchange their current perceptions about neighboring areas, and any vehicle evaluating conditions $(i)-(iv)$ uses the average value of all sensing vehicles in order to define its current perception about an area. Note that such an approach generally requires a fast and synchronized communication network. Hence, we define an asynchronous agreement algorithm, where those vehicles able to measure the suitability of area $i$ try to reach a common value by exchanging their perceptions and combining them by forming convex combinations. Figure 4 shows an example of the typical different IFD realizations for these two strategies and the no-cooperative sensing case (i.e., where vehicles just use their own perception to evaluate conditions $(i)-(iv)$). Note that the ultimate distribution has less variation when cooperative sensing is used. We have also run Monte Carlo simulations that show that when the ultimate distribution has less variation vehicles require more time to achieve it.

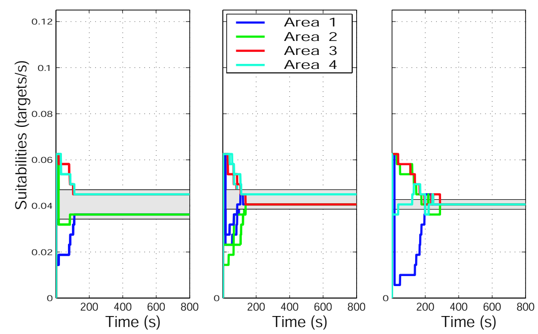


Fig. 4. Effects of implementing a synchronous and partially asynchronous iterative methods to try to reduce the effects of the sensing noise $w$ on the mission performance with 20 vehicles; no cooperative sensing (left), agreement strategy (middle), averaging strategy (right).

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