## Lecture 2

#### **Textbooks**

Main textbook: The Art of Hybrid Systems robotics.eecs.berkeley.edu/~sastry/ee291e/book.pdf

Good complementary lecture notes (chp. 1 - 4) robotics.eecs.berkeley.edu/~sastry/ee291e/ lygeros.pdf

#### Other textbooks:

A. van der Schaft and H. Schumacher. An Introduction to Hybrid Dynamical Systems. Lecture Notes in Control and Information Sciences 251, Springer-Verlag, 2000.

D. Liberzon. Switching in Systems and Control. Systems & Control: Foundations and Applications series. Birkhauser, Boston, 2003.

## Evaluate the following advice...



Stefan Sagmeister

Be careful whose advice you take.

Be passionate about your education. Work on finding your passions.

View work as a hobby. Enjoy it (it's your hobby after all).

Don't strive for success. Avoid mediocracy.

Worrying solves nothing<sup>†</sup>. Act or forget.

Don't waste time on jealousy. Everybody who is honest is interesting<sup>†</sup>

Money does not make me happy. Neither does makeup.

Trying to look good limits my life<sup>†</sup>. Appreciate and enjoy your body.

Everybody thinks they are right<sup>†</sup>.

Traveling opens new perspectives. Travel alone (once in a while).

Learn a foreign language. Learn think mathematically.

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## Inflatable monkey in Scotland (Stefan Sagmeister)





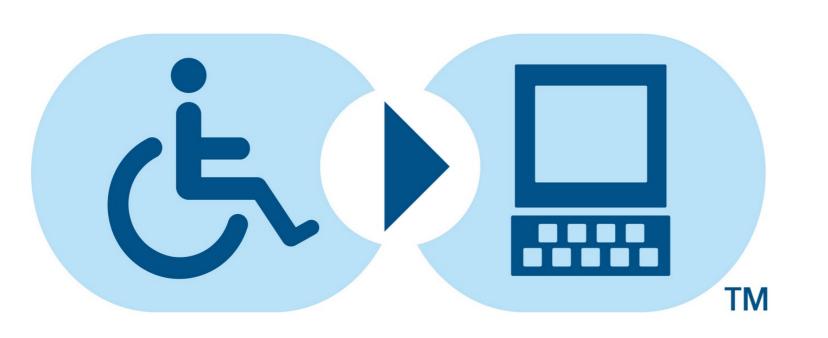








# ...from an engineering/science perspective...



Collect data +
Develop models

# If we have data, let's look at data. If all we have are opinions, let's go with mine.

- Jim Barksdale, former Netscape CEO

## In God we trust; all others must bring data

- W. Edwards Deming, statistician, professor, author, and consultant

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Trying to look good limits my life<sup>†</sup>. Appreciate and enjoy your body.

Everybody thinks they are right<sup>†</sup>. Use technology and learn from data + models.

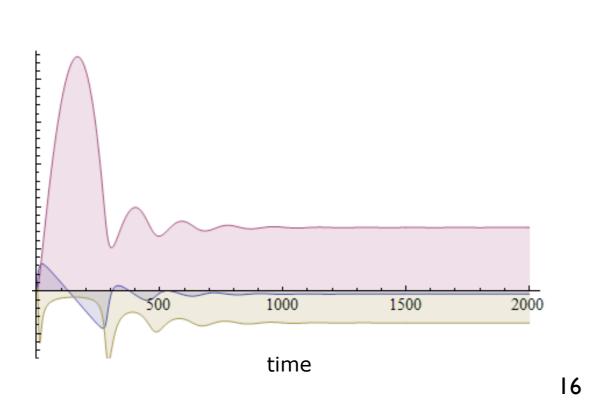
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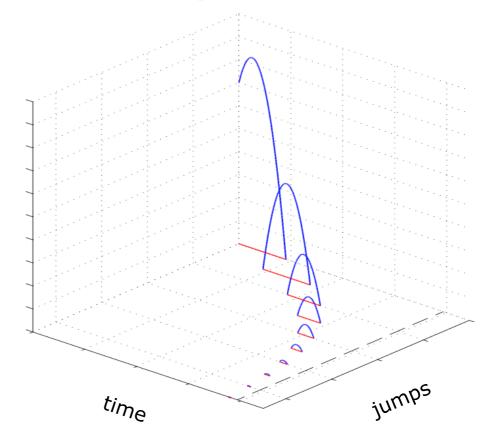
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#### Review

- State of the art modeling technique (characterize data on more sophisticated phenomena)
- Real love is hybrid! Stable? No! Why? Continuous + discrete dynamics.
- Other examples of hybrid systems
- Reality is less smooth than a set of differential eqs
- Notation, classification of dynamical systems

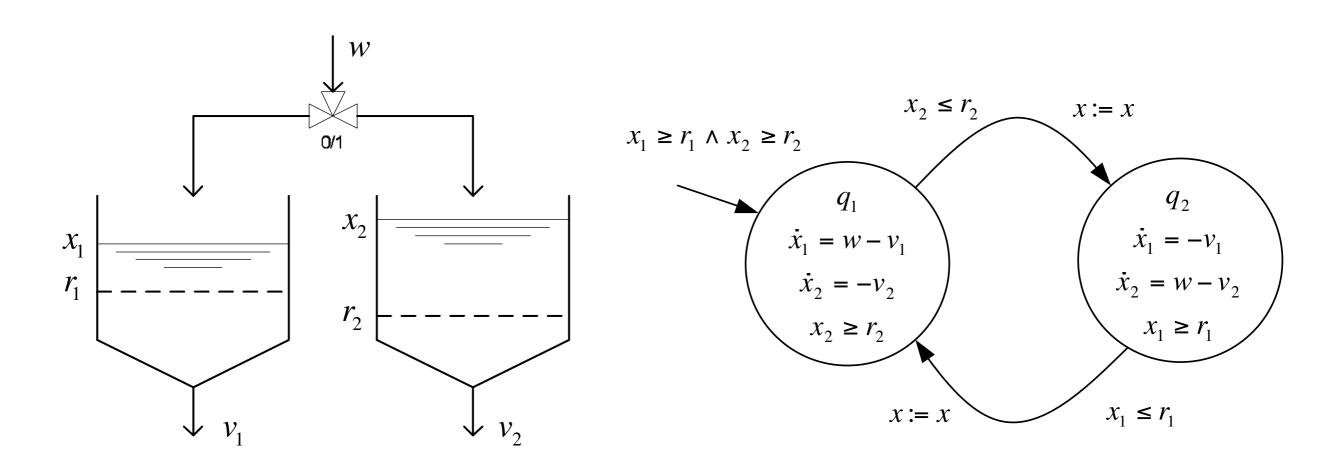


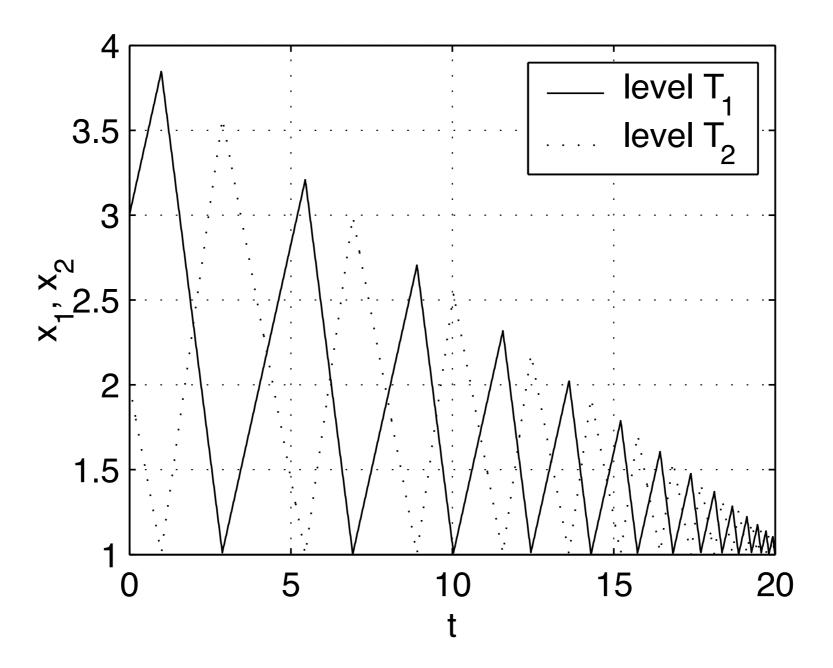


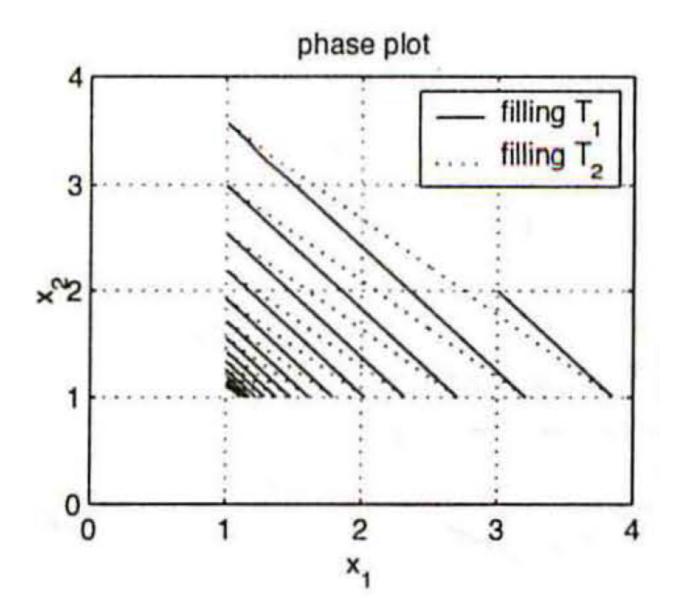
## Today

- Hybrid automata (deterministic case)
  - Definitions
  - Time sets + Executions
- Classification of solutions
- Next class: existence of solutions

## Tank example







# Simulating Hybrid Systems

#### MATLAB's Simulink

- suitable for a small number of discrete modes
- difficult to recover hybrid model

#### - Stateflow

- good for large numbers of discrete modes and complex transitions
- poor integration between continuous and discrete

#### - SHIFT

very good semantics (easily understandable), poor numerical algorithms

#### Modelica

- very good numerical algorithms
- very convenient for large models with interconnected components
- difficult to recover hybrid model

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## MATLAB's Simulink

## Simulink

$$\dot{x} = f(x) \qquad x(0) = x_0$$

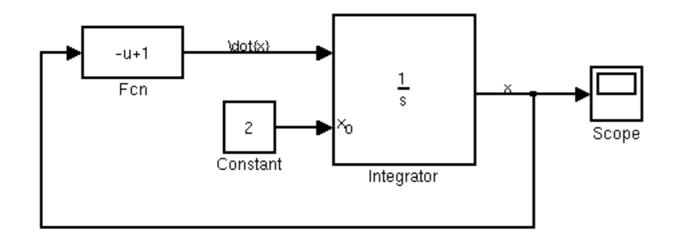
1. What you see: graphical user interface to build models of dynamical systems

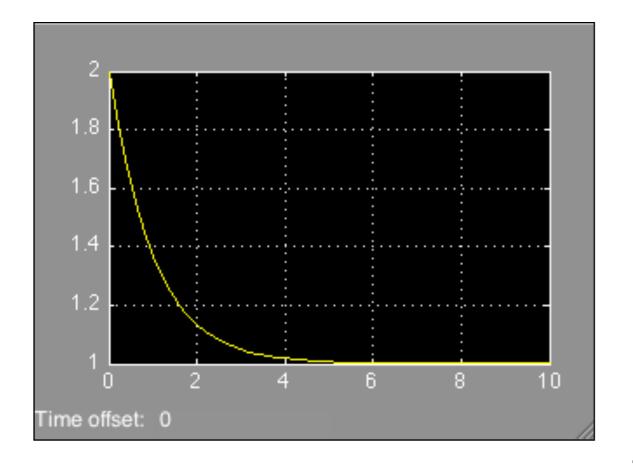
$$x(t) = x_0 + \int_0^t f(x(\tau)) d\tau \quad \Leftrightarrow \quad \boxed{ f(x) \quad \frac{x}{x_0} } \frac{1}{s}$$

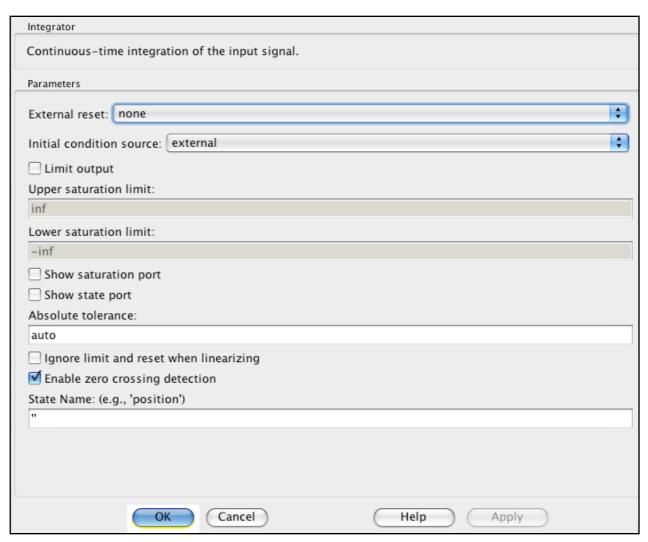
2. What's behind: numerical solver of ODEs with zero-crossing detection

## ODEs with initial conditions

$$\dot{x} = -x + 1 
x(0) = 2$$

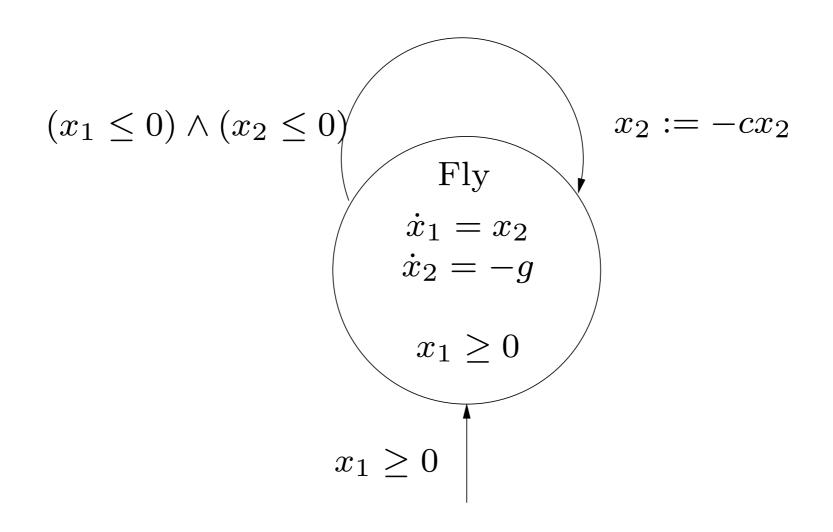




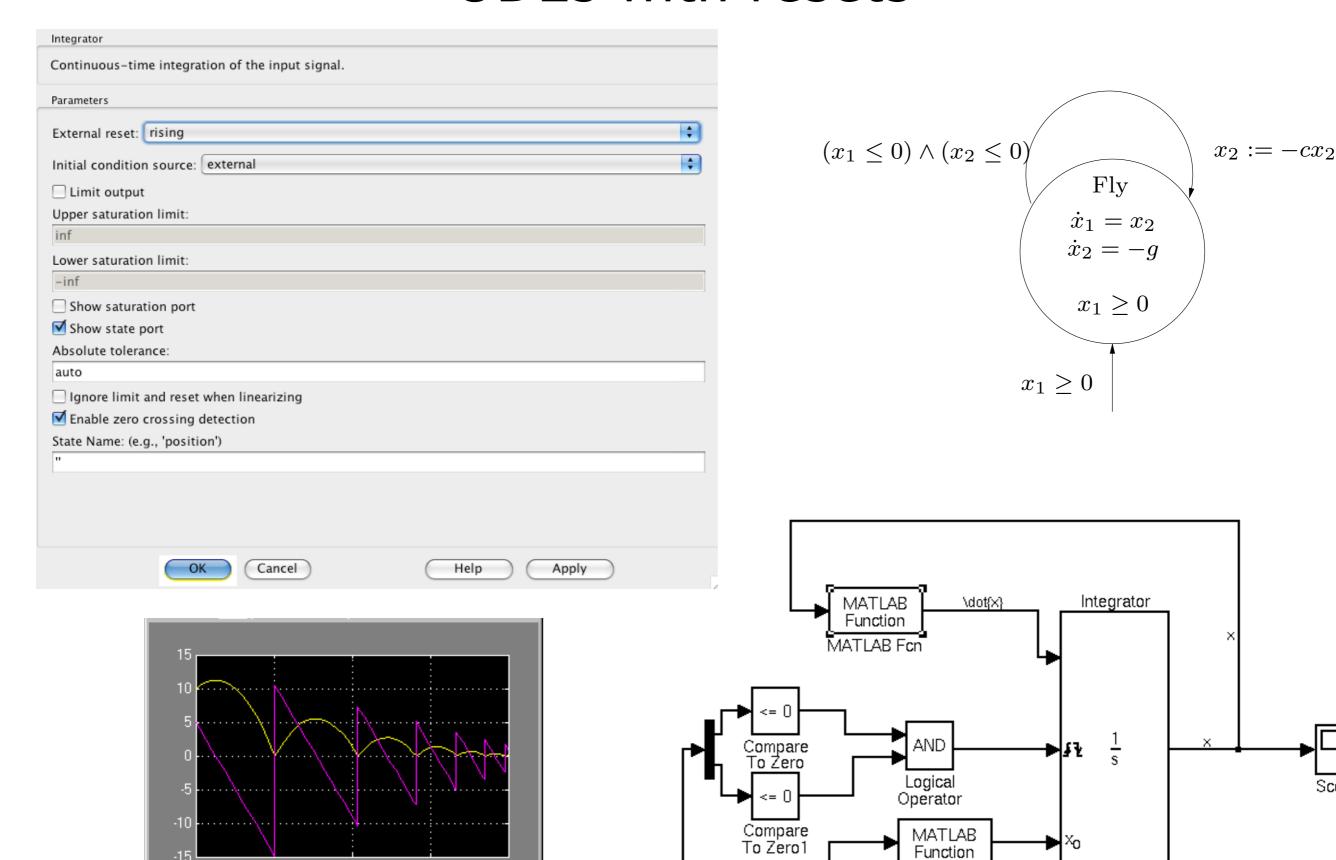


Fcn	
General expression block. Use "u" as the input variable name Example: $sin(u[1] * exp(2.3 * -u[2]))$	
Parameters	
Expression:  -u+1  Sample time (-1 for inherited):	
-1	
OK Cancel	Help Apply

## Bouncing ball



### ODEs with resets



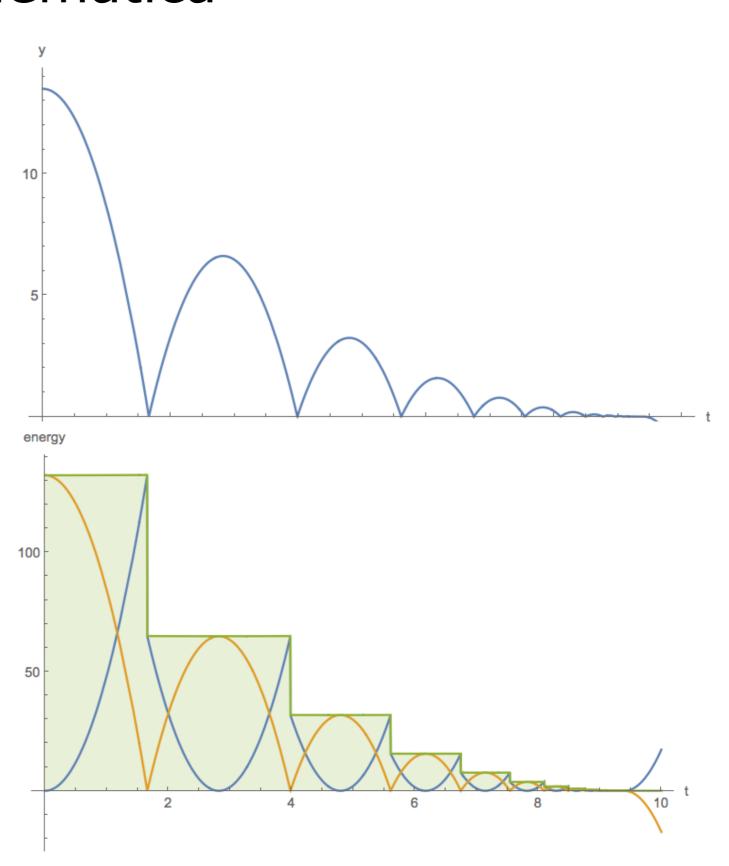
Time offset: 0

MATLAB Fcn1

Scope

#### Mathematica

```
imagesize = 500;
ball =
  NDSolve [ \{ y''[t] = -9.81, y[0] = 13.5, 
     y'[0] == 0, WhenEvent[y[t] == 0,
      y'[t] \rightarrow -0.7 y'[t], y,
    {t, 0, 10}];
kin[v_{-}] := .5 v^2;
pot[y_] := 9.8 y;
energy[y_{-}, v_{-}] := kin[v] + pot[y];
GraphicsGrid[
 {{Plot[y[t] /.ball, {t, 0, 10},
     AxesLabel \rightarrow {"t", "y"},
     ImageSize → imagesize],
   Plot [Evaluate [
      {kin[y'[t]], pot[y[t]],
         energy[y[t], y'[t]]} /. ball],
     \{t, 0, 10\}, Filling \rightarrow \{3 \rightarrow 0\},\
     AxesLabel → {"t", "energy"}]}}]
```



### What can go wrong?

- Problems in the continuous evolution
  - existence
  - uniqueness
  - finite escape
- Problems in the hybrid execution:
  - chattering
  - zeno
- Non-continuous dependency on initial conditions

...next class!