Rear Wheel Bicycle

Objectives

To analyse the dynamics of a bicycle with rear wheel steering and to compare the performance of two control schemes, a feedback controller and a PID controller. You should develop a model of the bicycle based on the guideline bellow from your text book.

Theoretical Introduction

Viewed as a dynamic system the controllability properties of a moving bicycle depend on the feedback mechanism created by its design, especially the position of its wheel steering (i.e. front versus real wheel steering). Getting a detailed model is often elaborated because the system has served degrees of freedom. For this exercise you will estimate an approximation based on a simple models.

"To derive the equations of motion we assume that the bicycle rolls on the horizontal xy plane. Introduce a coordinate system that is fixed to the bicycle with the ξ through the contact points of the wheels with the ground, the η -axis horizontal and the ζ -axis vertical, as shown in Figure 1. Let v_0 be the velocity of the bicycle at the rear wheel, b the wheel base, φ the tilt angle and δ the steering angle. The coordinate system rotates around the point O with the angular velocity $\omega = v_0\ \delta/b$, and an observer fixed to the bicycle experiences forces due to the motion of the coordinate system.

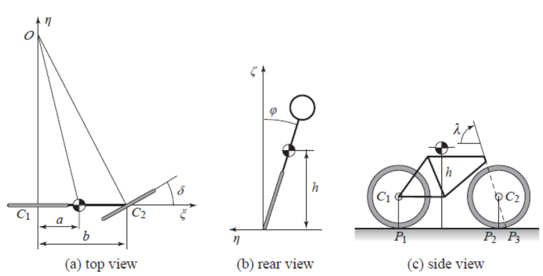

Figure 1: Schematic views of a bicycle

Let m be the total mass of the system, J the moment of inertia of this body with respect to the ξ -axis and $D = mah$ the product of inertia with respect to the ξ ζaxes. Furthermore, let the ξ and ζcoordinates of the center of mass with respect to the rear wheel contact point, P_1, be a and h, respectively. We have $J = mh^2$ and D = mah. The torques acting on the system are due to gravity and centripetal action. Assuming that the steering angle δ is small,the equation of motion becomes:

$$J\frac{d^2\varphi}{dt^2} - \frac{Dv_0}{b}\frac{d\delta}{dt} = mghsin(\varphi) + \frac{m{v_0}^2h}{b}\delta \tag{1}$$

The term $mghsin(\varphi)$ is the torque generated by gravity. The terms containing δ and its derivative are the torques generated by steering, with the term $\frac{Dv_0}{b}\frac{d\delta}{dt}$ due to inertial forces and the term $\frac{m{v_0}^2h}{b}\delta$ due to centripetal forces.

The design of the structural system has a big impact in the analysis of the bicycle dynamics, because of that the torque and the inclination are defined and applied to the handlebar and the frame in the control variable". [1]

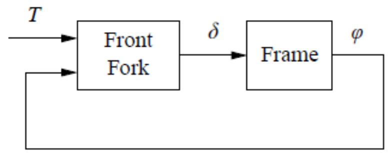

Figure 2: Block diagram of the bicycle with a front wheel steering

Under certain conditions, the feedback can actually stabilize the bicycle. A crude empirical model is obtained by assuming that the block B can be modelled as the static system.

$$\delta = k_1(V)T - k_2(V)\varphi \tag{2}$$

Procedure

- Make a detailed description of the aspects and characteristics used for the development of the model. (you must calculate the values of a, b, h,m)
- [0.5 points] Why is stability is important?
- [0.5 points] How many equilibrium points does your model have?
- [0.5 points] For each equilibrium point, using the Lyapunov theorem, determine its stability properties. Why is it important to prove that the equilibrium points are stable?
- [0.5 points] If there are unstable equilibrium points, how can you stabilize the system?, how can you controlled it? Make a quantitative and qualitative analysis.
- [0.5 points] Is your system reachable? Justify your answer.
- [1 point] Implement a feedback controller.
- [1.5 point] Implement a PID controller, considering that the stable state error must be equal to zero (if it is possible), the response time must be 5% faster than the close loop time.
- Find, for each controller:
  (a) Peak time
  (b) Maximum overshoot
  (c) Settling time

Note: Remember that the report **MUST** include:

- The results/findings of the lab
- Simulations of the parameters of the model

[1] Taken from `http://www.cds.caltech.edu/~murray/books/AM08/pdf/am08-complete_28Sep12.pdf`

- Simulations of the performance of both controllers.
- Simulink diagrams and Figures.
- Comparison between the experience of driving the bicycle and the simulations with both controllers.
- Conclusions
- References

References

1. Karl Johan Astrom, and Richard M. Murray. Feedback Systems: An Introduction for Scientist and Engineer. 2nd Edition. 2012, Princeton University Press.
2. Karl Johan Astrom, Richard E. Klein, and Anders Lennartsson. Bicycle Dynamics and Control: Adapted bicycles for education and research. Control Systems Magazine, IEEE, 2005