

## Objectives

1. To deepen in reachability and observability concepts.
2. To determine reachability and observability from state space models.

## Pre-lab: Aircraft

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. However, under certain assumptions, they can be decoupled and linearized into longitudinal and lateral equations. Aircraft pitch is governed by the longitudinal dynamics. In this example we will design an autopilot that controls the pitch of an aircraft. The basic coordinate axes and forces acting on an aircraft are shown in the figure given below. We will assume that the aircraft is in steady-cruise at

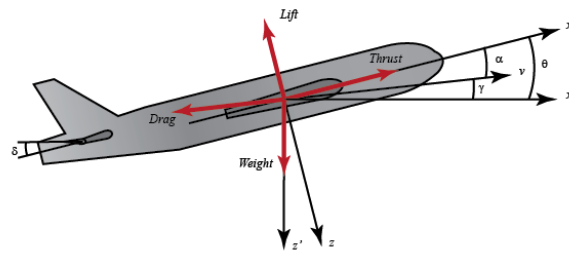


Figure 1: Forces acting on an aircraft

constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x and y directions. We will also assume that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifiesthe problem a bit). Under these assumptions, the longitudinal equations of motion for the aircraft can be written as follows.

$$\dot{\alpha} = \mu\Omega\sigma[-(C_L + C_D)\alpha + \frac{1}{(\mu - C_L)}q - (C_W \sin \gamma)\theta + C_L]$$

$$\dot{q} = \frac{\mu\Omega}{2i_{yy}}[[C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L)]q + (\eta C_W \sin \gamma)\delta]$$

$$\dot{\theta} = \Omega q$$

For this system, the input will be the elevator deflection angle  $\delta$  and the output will be the pitch angle  $\theta$  of the aircraft. Let's plug in some numerical values to simplify the modeling equations as is shown above:

$$\dot{\alpha} = -0.313\alpha + 56.7q + 0.232\delta$$

$$\dot{q} = -0.0139\alpha - 0.426q + 0.0203\delta$$

$$\dot{\theta} = 56.7q$$

Procedure:

- Verify if the system is stable or not.

- Design a State Estimator.
- Design a feedback controller. Place the poles at  $s_1 = -1, s_2 = -2, s_3 = -3$ .
- Design a feedback controller, where the settling time is less than the controller design before.
- Design a feedback controller, where the settling time is larger than the controller design before.

## Lab: Motor speed

A common actuator in control systems is the DC motor. It directly provides rotary motion and, coupled with wheels or drums and cables, can provide translational motion. The electric equivalent circuit of the armature and the free-body diagram of the rotor are shown in the following figure. For this example, we

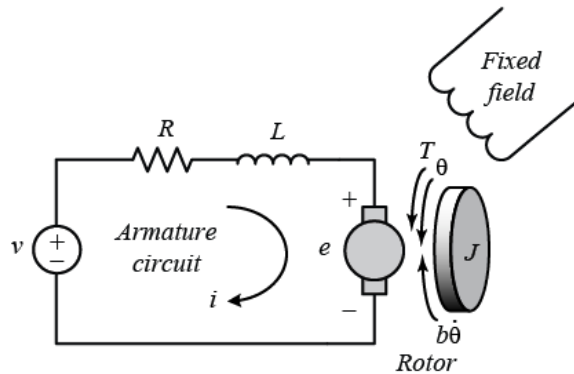


Figure 2: Motor

will assume that the input of the system is the voltage source ( $V$ ) applied to the motor's armature, while the output is the rotational speed of the shaft  $d(\theta)/dt$ . The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to shaft angular velocity.

The physical parameters for our example are:

- moment of inertia of the rotor ( $j$ ):  $0.01kgm^2/s^2$ .
- Damping ratio of the mechanical system ( $b$ ):  $0.1Nms$ .
- Electromotive force constant( $K = Ke = Kt$ ):  $0.01Nm/Amp$ .
- Electric Resistance ( $R$ ):  $1ohm$ .
- Electric inductance ( $L$ ):  $0.5H$ .
- The rotor and the shaft are assumed to be rigid.

Procedure:

- [1 point] Obtain the dynamic equation of the DC motor speed show in Figure 2.
- [1 point] Show the open-loop response of the system.
- [1 point] Calculate the eigenvalues of  $A$ . Is the system stable, asymptotically stable or unstable ?
- [2 points] Show that the system is controllable and observable. Show the procedure.

- [2 points] If the system is observable, design a state estimator (show the procedure). Show that the estimated states are a good approximation.
- [2 points] If the system is controllable, design a feedback controller (show the procedure). Make a detailed description of the parameters (e.g., poles) used in the design of the controller. Show the output of the system simulating with initial conditions.
- [1 point] Design using Mathematica a feedback controller, where the settling time is less than 2 seconds, and the overshoot is less than 5%. Show the output of the system, simulating with an unit step input (Remember: the input has to be multiply by  $kr$ ).

**Note:** Remember that the report **MUST** include conclusions.

- Detailed procedure.
- Answer to all questions.
- Send an email with the Notebook that you used.
- Conclusions.
- References.

## 0.1 References

1. Franklin, Gene F., Powell, David J. and Emami-Naeini Abbas. Feedback Control of Dynamic Systems. 5th Edition. 2006, Prentice-Hall Inc.
2. EE128 Fall 2007 Lab 1 guide. University of California, Bekerly. Online at <http://inst.eecs.berkeley.edu/ee128/fa07>
3. Control tutorial for Matlab and Simulink. Online at: <http://www.library.cmu.edu/ctms/ctms/basic/basic.htm>