

## Objectives

1. To be able to determine reachability and observability from state space models.
2. To practice the design of state estimators and feedback controllers.
3. To verify the effect of feedback controllers on a system.

## Pre-lab - Aircraft

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. However, under certain assumptions, they can be decoupled and linearized into longitudinal and lateral equations. Aircraft pitch is governed by the longitudinal dynamics. In this example we will design an autopilot that controls the pitch of an aircraft. The basic coordinate axes and forces acting on an aircraft are shown in Fig. 1.

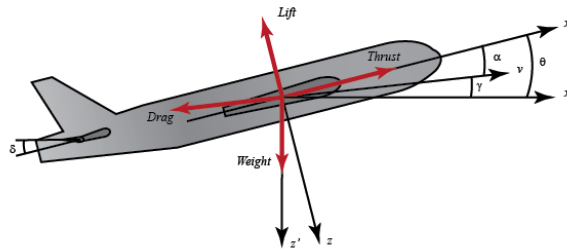


Figure 1: Forces acting on an aircraft. [4]

We will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the  $x$  and  $y$  directions. We will also assume that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic, but simplifies the problem a bit). Under these assumptions, the longitudinal equations of motion for the aircraft can be written as follows:

$$\begin{aligned}\dot{\alpha} &= \mu\Omega\sigma[-(C_L + C_D)\alpha + \frac{1}{(\mu - C_L)}q - (C_W \sin \gamma)\theta + C_L] \\ \dot{q} &= \frac{\mu\Omega}{2i_{yy}}[[C_M - \eta(C_L + C_D)]\alpha + [C_M + \sigma C_M(1 - \mu C_L)]q + (\eta C_W \sin \gamma)\delta] \\ \dot{\theta} &= \Omega q\end{aligned}$$

For this system, the input will be the elevator deflection angle  $\delta$  and the output will be the pitch angle  $\theta$  of the aircraft. Let's plug in some numerical values to simplify the modelling equations:

$$\begin{aligned}\dot{\alpha} &= -0.313\alpha + 56.7q + 0.232\delta \\ \dot{q} &= -0.0139\alpha - 0.426q + 0.0203\delta \\ \dot{\theta} &= 56.7q\end{aligned}$$

## Procedure

According to the model suggested before:

- Characterize the stability properties of each equilibrium point.
- Design a state estimator for the system.
- Design a feedback controller for the pitch angle. Place the poles at  $\lambda_1 = -1$ ,  $\lambda_2 = -2$ ,  $\lambda_3 = -3$ .
- Design a feedback controller where the settling time is less than in the previous one.
- Design a feedback controller where the settling time is greater than in the first controller.

## Problem 1 - Motor Speed

A common actuator in control systems is the DC motor. It provides rotary motion and, coupled with wheels or cable reels, can provide translational motion. The free-body diagram of the rotor and the equivalent electric circuit for the armature are shown in Fig. 2.

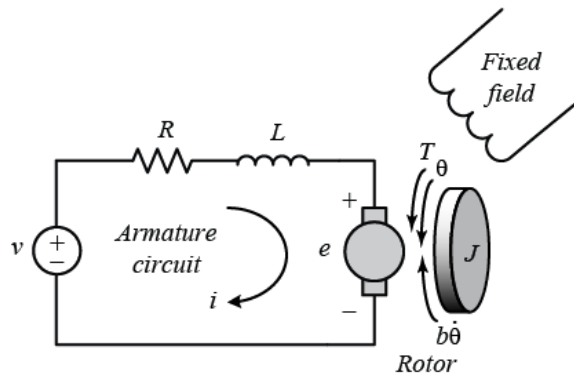


Figure 2: DC motor. [5]

For this example, we will assume that the input of the system is the voltage source ( $V$ ) applied to the motor's armature, while the output is the rotational speed of the shaft  $d(\theta)/dt$ . The rotor and shaft are assumed to be rigid. We further assume a viscous friction model, that is, the friction torque is proportional to the shaft angular velocity.

## Procedure

Consider the following parameters:

- Moment of inertia of the rotor  $J = 0.01 \text{ kg} \cdot \text{m}^2 / \text{s}^2$ .
- Damping ratio of the mechanical system  $b = 0.1 \text{ N} \cdot \text{m} \cdot \text{s}$ .
- Electromotive force constant  $K = K_e = K_t = 0.01 \text{ N} \cdot \text{m} / \text{A}$ .
- Electric resistance  $R = 1 \Omega$ .
- Electric inductance  $L = 0.5 \text{ H}$ .

Based on the previous explanation:

- [1 point] Obtain the dynamic equation of the speed of the DC motor shown in Fig. 2.
- [1 point] Using *Mathematica*, show the open-loop response of the system.
- [1 point] Calculate the eigenvalues of the  $A$  matrix. Is the system stable, asymptotically stable or unstable?
- [2 points] Verify if the system is controllable and observable. Show the procedure.
- [2 points] If the system is observable, design a state estimator for it and show the procedure. Show that the estimated states are a good approximation.
- [2 points] If the system is controllable, design a feedback controller and show the procedure. Make a detailed description of the parameters (e.g., poles) used in the design of the controller. Show the output of the system by simulating with initial conditions different to the origin.
- [1 point] Using *Mathematica*, design a feedback controller where the settling time is less than 2 seconds, and the overshoot is less than 5%. Show the output of the system by simulating with an unit step input. Remember that the input has to be multiplied by  $kr = -1/C(A - BK)^{-1}B$  (for more details, refer to section 7.2 of Åström [1]).

### General Remarks:

Remember that the report must include:

- A mathematical model (state space).
- A procedure.
- The answer to all the above questions.
- Conclusions.
- References.

Finally, please send the teaching assistant an email with the *Mathematica* Notebook that you used.

## References

- [1] K. Åström and R. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2nd ed, Princeton University Press, 2016.
- [2] G. Franklin, D. Powell and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 5th ed, Prentice-Hall Inc., 2005.
- [3] “EE128 Fall 2007 Lab 1 guide”, *University of California, Berkeley*. [Online]. Available: <http://www.library.cmu.edu/ctms/ctms/basic/basic.htm> [Accessed: 20-Sep-2017].
- [4] “Aircraft Pitch: System Modeling”, *Control Tutorials for MATLAB and Simulink*. [Online]. Available: <http://ctms.engin.umich.edu/CTMS/index.php?example=AircraftPitch&section=SystemModeling> [Accessed: 20-Sep-2017].
- [5] “DC Motor Speed: System Modeling”, *Control Tutorials for MATLAB and Simulink*. [Online]. Available: <http://ctms.engin.umich.edu/CTMS/index.php?example=MotorSpeed&section=SystemModeling> [Accessed: 20-Sep-2017].