Objective

To learn how to use Mathematica to run systems analysis.

Pre-lab: The car and the pendulum

The car with an inverted pendulum, shown below, is "bumped" with an impulse force, F . It is assumed that the motion takes place in a vertical plane, see Fig. 1.

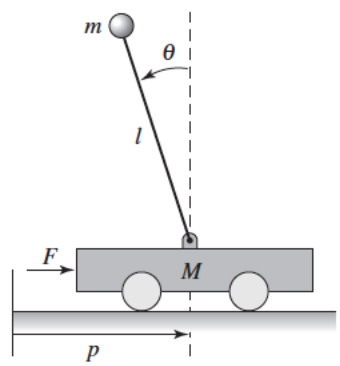

Figure 1: Car and pendulum

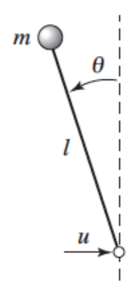

Figure 2: Pendulum

If we are interested only in analysing a pendulum's upright orientation without worrying about the location of base of the car (Fig. 2), the dynamics of this simplified system are given by

$$(I + ml^2)\ddot{\theta} - mglsin\theta = lucos\theta - \gamma\dot{\theta}$$

- m mass of the pendulum 0.2 kg
- γ friction of the car $0.6N/m/sec$
- l length to pendulum central 0.3 m

- I inertia of the pendulum 0.6 $kg * m^2$
- u force applied at the base
- θ pendulum angle from vertical

According to the model,

- Derive the input/state/output equations; i.e., write the equations in the form

$$\frac{dx}{dt} = f(x; u), y = h(x)$$

- Find all equilibrium solutions. Explain what physically represents each point.
- Plot the phase-portrait of the system and characterize the stability properties of each equilibrium point.

Also, an inverted pendulum can be described by the following normalized model

$$\frac{dx_1}{dt} = x_2, \quad \frac{dx_2}{dt} = sinx_1 + ucosx_1$$

Choosing the feedback law $u = -2sinx_1 - x_2cosx_1$, show that the equilibrium point (0,0) is locally stable.

Problem 1: Predator-Prey (Lotka-Volterra)

Consider a pair of species, one of which consists of predators whose population is denoted by y and the other its prey with population x. The Lotka-Volterra system which describes their behaviors is

$$\dot{x} = xa - bxy$$
$$\dot{y} = -cy + dxy$$

Where $a = 10$, $b = 2$, $c = 30$ and $d = 3$.

[1 point] Using figures, explain the evolution of the populations over time.

[1 point] Find all equilibrium solutions. Plot the phase-portrait of the system and characterize the stability properties of each equilibrium point. Explain the result.

Problem 2

Consider a system described by

$$\frac{dx_1}{dt} = x_2 + x_1(1 - x_1^2 - x_2^2), \quad \frac{dx_2}{dt} = -x_1 + x_2(1 - x_1^2 - x_2^2)$$

- [0.5 points] Plot the phase-portrait of the system and characterize the stability properties of the equilibrium points.
- [1 point] Illustrate the Lyapunov's concept of a stable solution using different initial conditions and comparing its response with the equilibrium.

Problem 3

Consider a system described by

$$\dot{x}(t) = Ax(t)$$

where

$$\dot{x_1} = -x_1 + 4x_2$$
$$\dot{x_2} = -x_1 - x_2^3$$

and the Lyapunov function

$$V = x_1^2 + ax_2^2$$

- [0.5 points] How many equilibrium points does the system have?
- [1 point] Using the Lyapunov stability theorem, For which value of a can you state that the system is globally asymptotically stable? Verify your result with a simulation.

Note: Remember that the report must include:

- Mathematical Model (state space).
- Procedure.
- Answer to all questions.
- Send an email with the Notebook that you used.
- Conclusions.
- References.