

## Objective

1. To learn how to use *Mathematica* to run systems analysis.
2. To understand some of the basic concepts behind the control theory.

## Problem 1 - Insulin-Glucose dynamics

The concept of *feedback* is present in many dynamical systems. One of the most famous examples is the regulation of the glucose in the blood-stream. The glucose concentration is influenced by different factors as shown in Fig. 1. The body attempts to maintain a constant concentration of glucose through the production of insulin and glucagon. When glucose levels rise, the pancreas secretes insulin and causes the body to store excess glucose in the liver. When glucose levels fall, the hormone glucagon is produced by the pancreas, which has the opposite effect. For more details, read section 4.6 of Åström [1] and watch the video available at [https://www.youtube.com/watch?v=ae\\_jC4FD0Uc](https://www.youtube.com/watch?v=ae_jC4FD0Uc).

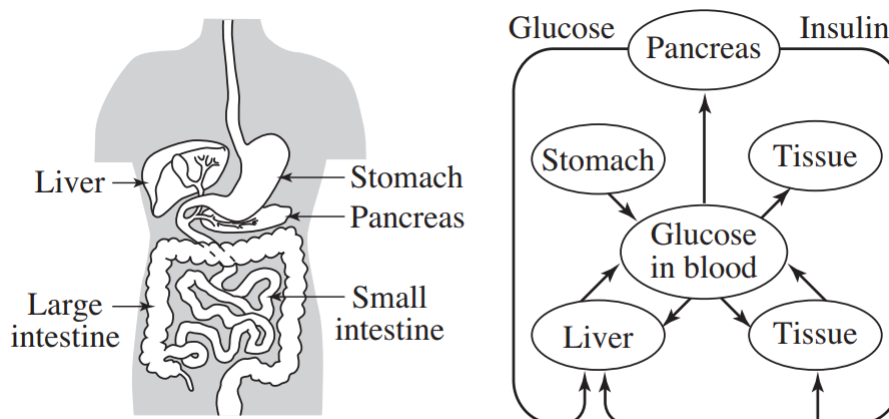


Figure 1: Insulin-glucose dynamics. [1]

## Procedure

According to the model suggested in Åström [1]:

- [0.5 points] Illustrate in a block diagram the system that regulates the levels of glucose in the human body.
- [1 point] Using *Mathematica*, simulate the model. In particular, show and explain the evolution of the concentration of glucose and interstitial insulin. Let the variable  $x = [x_1, x_2]^T$  define the state of the system, where  $x_1$  represents the concentration of glucose and  $x_2$  is proportional to the concentration of **interstitial** insulin. Consider the following parameters:
  - (a) Constants  $p_1 = 0.0264$ ,  $p_2 = 0.0254$ , and  $p_3 = 0.000012$

- (b) An equilibrium value of glucose  $x_1^e = 90mg/dl$ .
  - (c) An equilibrium value of insulin  $x_2^e = 7uU/ml$
  - (d) An initial value of glucose  $x_1(0) = 280mg/dl$ .
- [0.5 points] Plot the phase-diagram of the system around the point  $(90, 0)$  (use the function *StreamPlot* in *Mathematica*). Assume the input  $u = x_2^e$ . What can you conclude from the plot?
  - [1 point] Diabetes is a disease which affects many people around the world. Which values should you change in the model to simulate diabetes? Your answer have to be supported through simulations. Using the new model, how does the behavior of the level of glucose change if you increase the input (level of insulin in the **bloodstream**)? Illustrate your answer using simulations.

## Problem 2 - Mass-Spring system

Consider the Mass-Spring system shown in Fig. 2, where  $F_s(x)$  is the spring force,  $F_f(\dot{x})$  is the friction coefficient,  $x(t)$  is the displacement of the mass and  $F_a(t)$  the applied force:

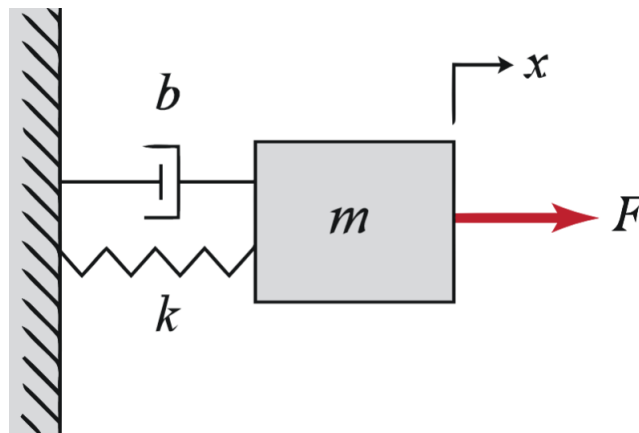


Figure 2: Mass-Spring System. [3]

Note that for a displacement of the mass of  $x(t)$ , the speed

$$v = \frac{dx(t)}{dt}$$

and the acceleration

$$a = \frac{dv(t)}{dt}$$

Now, consider

$$F_f(v) = Bv$$

$$F_s(x) = Kx(t)$$

where  $B$  is the friction coefficient and  $K$  the spring constant.

## Procedure

- [1 point] Using Newton's Law, obtain the dynamic equations of the Mass-Spring system model shown in Figure 2. Consider the following parameters:
  - (a) A friction coefficient  $B = 30Nsec/m$ .
  - (b) A spring constant  $K = 15N/m$ .
  - (c) A mass  $M = 700kg$ .
  - (d) A force  $a$  as the input,  $300N$ .

Present a detailed procedure to find the mathematical model, that is the state space representation.

- [1 point] Using *Mathematica*, simulate the model considering two different inputs of your choice. Make a detailed description of every input you use, associating it to a real situation. Describe the behavior of the model for each input.

## General Remarks:

Remember that the report must include:

- A mathematical model (state space).
- A procedure.
- The answer to all the above questions.
- Conclusions.
- References.

Finally, please send the teaching assistant an email with the *Mathematica* Notebook that you used.

## References

- [1] K. Åström and R. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2nd ed, Princeton University Press, 2016.
- [2] D. Kerner, "Minimal Models for Glucose and Insulin Kinetics", *Civilized Software Inc*, 2013.
- [3] "Introduction: System Modeling", *Control Tutorials for MATLAB and Simulink*. [Online]. Available: <http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=SystemModeling> [Accessed: 10- Aug- 2017].