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On the Stability of Resource Undermatching in Human Group-Choice

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Abstract— **The analysis of patterns of social interaction plays an important role in providing services on online platforms (e.g., in designing algorithms for the allocation of information resources). The proposed model takes into account human factors underlying the concept of the Ideal Free Distribution (IFD), which captures empirical patterns of the aggregate group-level behavior of individuals competing for resources. The model explains the phenomenon of resource undermatching as a natural IFD-based outcome resulting from boundedly rational decision-making (i.e., individuals perceive only some of the available resources). We show that undermatching can be described as a globally balanced state in which the perceived cost of the best forgone alternatives is approximately the same for all individuals. Furthermore, we identify conditions that guarantee stability. From this analysis, we infer that the matching of the aggregate of individual choices to resources is independent of their initial distribution. Finally, we quantify the effect of resource scarcity on the degree of matching.**

I. INTRODUCTION

Recent years have witnessed a tremendous growth in social mobility and online platforms on which service providers leverage large amounts of data for allocating resources. The analysis and design of cost-effective, customized user services requires a good understanding of the impact of information flow constraints on the overall behavior of these distributed systems (e.g., in a virtual marketplace where providers compete for a pool of customers [1]). Information foraging tries to explain how individuals seek, gather, and share information, relying solely on local mechanisms to make decisions [2]. It provides a behavioral framework for developing algorithms that effectively support key concepts underlying the formation of patterns of social interaction (e.g., in order to counterbalance inefficient allocations, it is key to understand the extent to which noisy information biases human decisions [3]). The concept of the Ideal Free Distribution (IFD) has recently received much attention in the information foraging literature and has been useful to describe the relationship between a limited set of resources distributed across certain options and the aggregate of individual choices (group-choice behavior) often found in empirical data [4]–[9].

According to the original IFD, group-choice approximates an equilibrium state where the number of resources per individual is the same for any option (i.e., the ratio between the number of resources associated to an option and the number of individuals selecting that option is the same for all individuals across all options). The word *ideal* means that all individuals are equally competitive and are rational (i.e., have complete information about the number of resources associated to and the number of individuals choosing any option). The word *free* means that individuals can make decisions instantaneously, without incurring an extra cost for changing options. Broadly speaking, IFD patterns of human decision-making suggest that there are mechanisms (incentives) producing well-defined aggregate behaviors. Understanding how these patterns are influenced by social and environmental factors is an important step in trying to incorporate broad social behavior into the design of algorithms underlying online services (e.g., algorithms for resource allocation).

Over the past fifteen years, social group-choice experiments have shown that individuals are less sensitive to the allocation of resources than predicted by the original IFD, a systemic deviation called resource undermatching [4]–[9]. Undermatching means that fewer than expected individuals select the most profitable option, while more individuals choose the least profitable one. Even when the rationality of individuals is limited to locally available resources and information, there exists a noticeable degree of undermatching [5]–[7].

This work introduces a bounded rationality model that can explain the phenomenon of resource undermatching as a natural outcome from human decision-making. It is closely related to the work in [10] where the authors propose a probabilistic model to evaluate how spatial limitations affect the expected distribution of choices. Our model, however, focuses on how the individuals' perceptions affect the degree of matching. It is built on an undirected network of options, which constrains the available resources. Most importantly, the model relaxes the ideal assumption of the original IFD, allowing us to characterize human incentives based solely on local information. Instead of assuming that individuals behave as if they were maximizing the number of resources per individual (like the work in [11]), individuals behave as if they were minimizing opportunity costs. The resulting outcome exhibits a degree of undermatching that does not depend on the variability of resources or the number of available options.

Our contribution is threefold. First, we propose a mechanism for decision-making based on the notion of opportunity cost for rationally bounded individuals (i.e., decisions depend on information about locally available alternatives). Second, we show that the IFD can be described as a globally balanced state, where the opportunity cost across options is approximately the same. Third, we present an analytical expression for the degree of matching for scenarios where there exists a small scarcity of resources (i.e., when the total

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number of individuals is slightly smaller than the number of resources).

The remainder of this paper is structured as follows. Section II describes the proposed model. Section III describes the phenomenon of undermatching as a globally balanced state. Section IV introduces the mechanism for decision-making and Section V shows that the set representing resource undermatching is invariant. Section VI presents conditions that guarantee that the invariant set is asymptotically stable. Section VII describes simulation results that describe the effects of resource scarcity on the degree of matching. Finally, Section VIII draws some conclusions and future research directions.

II. The model

Consider an undirected network $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} = \{1, \ldots, n\}$ represents the available options (nodes). We say that options i and j are adjacent (i.e.,$\{i, j\} \in \mathcal{A}$) if an individual is allowed to change option i for option j and vice versa. The neighborhood of option i is the set of all locally available options defined by $\mathcal{N}_i := \{j \in \mathcal{N} : \{i, j\} \in \mathcal{A}\}$. Individuals who select option $i \in \mathcal{N}$ may change options (select different nodes) without incurring an extra cost, but only according to the constraints imposed by $\mathcal{N}_i$.

Let $x_i \in \mathcal{R}, \mathcal{R} = \{1, \ldots, m\}$, describe the number of individuals deciding to choose option i; the constant $m = x_1 + \cdots + x_n$ denotes the total number. Define $\mathbf{x} = (x_1, \ldots, x_n)$ as the state of the system and let

$$\mathcal{X} := \left\{ \mathbf{x} \in \mathcal{R}^n : \sum_{i \in \mathcal{N}} x_i = m \right\}$$

Note that the state space $\mathcal{X}$ represents the set of all possible choices across the network of options $\mathcal{G}$. Moreover, let $w_i \in \mathbb{N}$ describes the number of local resources associated to option i.

Using the generalized habitat matching rule, undermatching can be described by

$$\log \frac{x_i}{x_j} = a \log \frac{w_i}{w_j} \tag{1}$$

where $a > 0$, known as the degree of matching, represents a measure of sensitivity between a group of individuals and a set of resources [7]. On a logarithmic scale, the distribution of choices is directly proportional to the number of resources. According to eq. (1), the value of the degree of matching can be described as

$$a = \frac{\log\left(\frac{x_i}{x_j}\right)}{\log\left(\frac{w_i}{w_j}\right)} \tag{2}$$

Next, we propose how a local decision-making mechanism may produce values that are consistent with the empirical range of a, $0.5 \leq a \leq 0.9$ [4]–[9].

Suppose that individuals make decisions based on perceptions, which in general depend on x_i and w_i. Conceptually, the value of $\varphi_i(x_i, w_i)$ represents the average assessment that individuals make of option i. The incentive to change option $i \in \mathcal{N}_j$ for option j depends on the relative perception difference between both options (i.e., the opportunity cost of choosing one option over another). In particular, consider the set of states

$$\mathcal{X}_u := \{\mathbf{x} \in \mathcal{X} : |\varphi_i(x_i, w_i) - \varphi_j(x_j, w_j)| \leq h, \forall i, j \in \mathcal{N}\} \tag{3}$$

where $h \geq 0$ is a constant. If $\mathbf{x} \in \mathcal{X}_u$, perceptions about any pair of options i and j are considered similar and the opportunity cost across any two options does not differ by more than h.

Let $w = w_1 + \cdots + w_n$ be the total number of resources available across the entire network. We require the following assumptions on the options and the resources for which individuals compete.

Assumption 1 (Availability): There exists some scarcity of resources, $r \in \mathbb{R}, 0 < r < 1$, such that $w = c(1-r)m$, where $c = 1$ is a constant with units of resources/individuals.

Assumption 2 (Variability): Any difference in the number of locally available resources is bounded by $|w_i - w_j| \leq \frac{w}{n}$ for all $i, j \in \mathcal{N}$.

Assumption 3 (Connectivity): The network of options $\mathcal{G}(\mathcal{N}, \mathcal{A})$ is connected, without self-loops or parallel edges.

Assumption 1 implies that, if each individual pursues one resource unit, there are not enough services to perfectly meet individuals' wants, thereby inducing competition. Assumption 2 places a bound on the difference between the number of resources associated to the different options. All options must be relatively attractive, despite the particular distribution of individual choices (i.e., there does not exits an option that carries most of the total of resources). Both, Assumptions 1 and 2 are key to determine the degree of matching. Finally, according to Assumption 3, individuals must be allowed to change any option for at least another.

III. Characterizing undermatching

Here, we focus on scenarios in which $r < 0.5$. Define the perception of option i as

$$\varphi_i(x_i, w_i) = cx_i - w_i \tag{4}$$

Eq. 4 means that an individual can perceive the difference in resources between the number of resources being pursued by the individuals selecting option i and the number of resources associated to that option. To determine the value of the degree of matching, note that if $h = 0$, $\mathbf{x} \in \mathcal{X}_u$ only when $cx_i - w_i = cx_j - w_j$ for all $i, j \in \mathcal{N}$. Because the perceptions must be equal for all options, we know that for any option $i \in \mathcal{N}$

$$n(cx_i - w_i) = c\sum_{j=1}^{n} x_j - \sum_{j=1}^{n} w_j = cm - w \tag{5}$$

Using eq. (5), the number of individuals selecting option i can be described as

$$x_i = \frac{m}{n} + \frac{1}{c}\left(w_i - \frac{w}{n}\right)$$

and according to Assumption 1

$$x_i = \frac{mr}{n} + \frac{w_i}{c} \tag{6}$$

Let $w_p = \max_i\{w_i\}$ and $w_q = \min_i\{w_i\}$ (i.e., options p and q represent the options with the most and the least available resources, respectively). Because each option satisfies eq. (6), applying eq. (2) with option $i = p$ and option $j = q$, we know that when $\mathbf{x} \in \mathcal{X}_u$

$$a = \frac{\log\left(\frac{cmr+nw_p}{cmr+nw_q}\right)}{\log\left(\frac{w_p}{w_q}\right)} = \frac{\log\left(\frac{w_p}{w_q} - \frac{cmr(w_p-w_q)}{w_q(cmr+nw_q)}\right)}{\log\left(\frac{w_p}{w_q}\right)} \tag{7}$$

Now, because $\frac{cmr(w_p-w_q)}{w_q(cmr+nw_q)} \geq 0$, the degree of matching is bounded by $a \leq 1$. In other words, if we define perceptions according to eq. (4), then the set $\mathcal{X}_u$ represents an IFD with undermatching. Equation (7) implies that when $h = 0$, the analytical value of the degree of matching is unique. There will always be some degree of undermatching, unless all options have the same number of available resources. Only when $w_p = w_q$, which implies that $w_i = w_j$ for all $i, j \in \mathcal{N}$, the set $\mathcal{X}_u$ represents an IFD with strict matching (i.e., for the particular case where resources are uniformly distributed). In general, the degree of undermatching depends on the options with the maximum and minimum number of resources (options p and q).

Next, according to eq. (6), when $\mathbf{x} \in \mathcal{X}_u$, there exists two bounds $\bar{x}_i^l$ and $\bar{x}_i^u$ such that any option i satisfies

$$\bar{x}_i^l = \left\lfloor \frac{mr}{n} \right\rfloor + \frac{w_i}{c} \leq x_i \leq \left\lceil \frac{mr}{n} \right\rceil + \frac{w_i}{c} = \bar{x}_i^u \tag{8}$$

Because $x_i \in \mathbb{N}$, the value of x_i must equal one of the bounds (i.e., if $\mathbf{x} \in \mathcal{X}_u$, then $x_i \in \{\bar{x}_i^u, \bar{x}_i^l\}$ for every $i \in \mathcal{N}$). Consider a state $\mathbf{x} \in \mathcal{X}_u$ and options i and j such that $\varphi_i(x_i, w_i) > \varphi_j(x_j, w_j)$ (i.e., the case when $h \neq 0$). The largest difference in perceptions between both options is

$$\begin{aligned}\varphi_i(x_i, w_i) - \varphi_j(x_j, w_j) &< \varphi_i(\bar{x}_i^u, w_i) - \varphi_j(\bar{x}_j^l, w_j) \\ &= (c\bar{x}_i^u - w_i) - (c\bar{x}_j^l - w_j) \\ &= c\left(\left\lceil \frac{mr}{n} \right\rceil - \left\lfloor \frac{mr}{n} \right\rfloor\right)\end{aligned}$$

In other words, given that $x_i \in \mathbb{N}$, the upper bound on the largest difference in perceptions depends on the total number of individuals, the level of resource shortage, and the number of available options. Because $\left\lceil \frac{mr}{n} \right\rceil - \left\lfloor \frac{mr}{n} \right\rfloor$ takes values of either 0 or 1, it is the combination of m, r, and n which determines whether two options are considered similar when their perceptions differ by one. Here, we restrict the maximum difference in perceptions when $\mathbf{x} \in \mathcal{X}_u$ to $h \in \{0, 1\}$.

Next, note that the options with the maximum and minimum number of resources (options p and q), each satisfies eq. (8). Moreover, using eq. (2) with option $i = p$ and option $j = q$, we can derive the following bounds on the degree of matching

$$\log\left(\frac{\left\lfloor \frac{mr}{n} \right\rfloor + \frac{w_p}{c}}{\left\lceil \frac{mr}{n} \right\rceil + \frac{w_q}{c}}\right) \leq a \log\left(\frac{w_p}{w_q}\right) \leq \log\left(\frac{\left\lceil \frac{mr}{n} \right\rceil + \frac{w_p}{c}}{\left\lfloor \frac{mr}{n} \right\rfloor + \frac{w_q}{c}}\right)$$

The value of $a \log\left(\frac{w_p}{w_q}\right)$ equals the lower bound, when according to eq. (8) $x_p = \bar{x}_p^l$ and $x_q = \bar{x}_q^u$; and the upper bound when $x_p = \bar{x}_p^u$ and $x_q = \bar{x}_q^l$.

IV. Dynamics of Choice

To propose a mechanism of how individuals reach an IFD with undermatching, let us define a discrete event system $S = (\mathcal{X}, \mathcal{G}, \mathcal{E}, g, f_e)$. The set $\mathcal{E}$ denotes all possible events underlying the dynamics of choice; an event at time index k is described as $e(k)$. Transitions between states depend on the activation function g. For $\mathbf{x}(k) \in \mathcal{X}$, we say that an event $e(k)$ is active, if $e(k) \in g(\mathbf{x}(k))$. If an active event $e(k)$ occurs, the transition function f_e generates the state $\mathbf{x}(k+1)$ defined by $\mathbf{x}(k+1) := f_{e(k)}(\mathbf{x}(k))$. Note that at time index k there is one current state, but different event sequences may lead to the same state (only one of possibly several active events may occur). If there is a deadlock, the only active event is the null event e^0, where $f_{e^0}(\mathbf{x}(k)) = \mathbf{x}(k)$.

To specify the set of events $\mathcal{E}$ and the transition function f_e, let

$$\mathcal{M}_i := \{j \in \mathcal{N}_i : \varphi_i(x_i, w_i) < \varphi_j(x_j, w_j)\} \tag{9}$$

be the set of options $j \in \mathcal{N}_i$ for which option i is a better local alternative. Based on eq. (9) we let individuals behave as if they were minimizing the opportunity costs associated to the various options (i.e., if $\varphi_i(x_i, w_i) - \varphi_j(x_j, w_j) < 0$ then option i is an attractive alternative over option j).

With e_{ji}, $j \in \mathcal{M}_i$, we represent an individual's decision to change option j for option i. Let $\mathcal{E}_\alpha = \{e_{ji}\}$ be the set of all possible switching decisions across the option network. Then, the set of events is given by the powerset of $\mathcal{E}_\alpha$ (without the empty set), $\mathcal{E} = P(\mathcal{E}_\alpha) \setminus \{\emptyset\}$, and an event $e(k) \in \mathcal{E}$ is a set where each element represents a decision to change options. We assume that no two individuals choose to change the same option simultaneously. Moreover, if e_{ji} captures the decision of one individual at time index k, and an active event $e(k)$ occurs with $e_{ji} \in e(k)$, then we consider that $e_{ji'} \notin e(k)$ for any $i' \neq i \in \mathcal{N}_j$.

To indicate whether an individual chooses another available option over option i, let

$$\mathbb{1}_i^-(k) = \begin{cases} 1, & \text{if } e_{ij} \in e(k) \text{ for some } j \in \mathcal{N}_i; \\ 0, & \text{otherwise} \end{cases}$$

Similarly, the indicator function $\mathbb{1}_i^+(k)$ indicates whether an individual finds option i more attractive than other locally available options, that is

$$\mathbb{1}_i^+(k) = \begin{cases} 1, & \text{if } e_{ji} \in e(k) \text{ for some } j \in \mathcal{N}_i; \\ 0, & \text{otherwise} \end{cases}$$

The number of individuals choosing option i at $k+1$ is

$$x_i(k+1) = x_i(k) - \mathbb{1}_i^-(k) + \mathbb{1}_i^+(k) \quad (10)$$

V. INVARIANCE OF IFD WITH UNDERMATCHING

Next, we will show that if $\mathbf{x}(k) \in \mathcal{X}_u$ for some $k \geq 0$, then $\mathbf{x}(k') \in \mathcal{X}_u$ for all $k' \geq k$ (i.e., the set $\mathcal{X}_u$ is invariant). If $\mathbf{x}(k) \in \mathcal{X}_u$, consider the two possible scenarios based on the value of h. If $h = 0$, we know that according to eq. (3), the perceptions about any two options at time instant k are equal. Then, according to eq. (9), the set $\mathcal{M}_i = \emptyset$ for all $i \in \mathcal{N}$, i.e., there are no incentives for individuals to change their current options; the only active event is e^0. We know that at time $k+1$, $x_i(k+1) = x_i(k)$ for all $i \in \mathcal{N}$. Thus, if $h = 0$ then $\mathbf{x}(k+1) \in \mathcal{X}_u$.

Now, if $h = 1$, according to eq. (3), the opportunity cost between options i and j at time k satisfies

$$|\varphi_i(x_i(k), w_i) - \varphi_j(x_j(k), w_j)| \leq 1$$

First, if the magnitude of the difference in perceptions is strictly less than one, then because $x_i, w_i \in \mathbb{N}$, $\varphi_i(x_i(k), w_i) = \varphi_j(x_j(k), w_j)$. Again, according to eq. (9), individuals will not change options. Second, if the magnitude is equal to one, without loss of generality, consider two options p and q such that $\varphi_p(x_p(k), w_p) > \varphi_q(x_q(k), w_q)$. Using eq. (10), the difference in perceptions between options p and q at a time $k+1$ is given by

$$\begin{aligned}
&|\varphi_p(x_p(k+1), w_p) - \varphi_q(x_q(k+1), w_q)| \\
&= |cx_p(k+1) - w_p - cx_q(k+1) + w_q| \\
&= |cx_p(k) - c\mathbb{1}_p^-(k) + c\mathbb{1}_p^+(k) - w_p - cx_q(k) + c\mathbb{1}_q^-(k) \\
&\quad - c\mathbb{1}_q^+(k) + w_q|
\end{aligned}$$

Because $h = 1$, then $x_p(k)$ is equal to the upper bound and $x_q(k)$ to the lower bound in eq. (8), i.e., $x_p(k) = \bar{x}_p^u$ and $x_q(k) = \bar{x}_q^l$. Option p is not attractive for any individual to choose because it has the highest perception possible. According to eq. (9), $\mathcal{M}_p = \emptyset$ and $\mathbb{1}_p^+(k) = 0$. Because $\varphi_q(x_q, w_q)$ is the lowest perception possible, individuals choosing option q will not change their option and $\mathbb{1}_q^-(k) = 0$. Thus

$$\begin{aligned}
&|\varphi_p(x_p(k+1), w_p) - \varphi_q(x_q(k+1), w_q)| \\
&= \left|cx_p(k) - c\mathbb{1}_p^-(k) - w_p - cx_q(k) - c\mathbb{1}_q^+(k) + w_q\right|
\end{aligned}$$

And because $cx_p(k) - w_p - cx_q(k) + w_q = 1$,

$$\begin{aligned}
&|\varphi_p(x_p(k+1), w_p) - \varphi_q(x_q(k+1), w_q)| \\
&= \left|1 - c\mathbb{1}_p^-(k) - c\mathbb{1}_q^+(k)\right| \leq 1
\end{aligned}$$

Therefore, $\mathbf{x}(k+1) \in \mathcal{X}_u$ and the set of states representing undermatching is invariant.

VI. STABILITY PROPERTIES OF THE MODEL

The following theorem establishes the stability properties of $\mathcal{X}_u$. It shows how the dynamics of the aggregate of individual choices ultimately undermatches the resources. The proof of the following theorem is presented in the Apendix.

Theorem 1: Suppose that Assumptions 1-3 hold. Then, the invariant set $\mathcal{X}_u$ has a region of asymptotic stability equal to $\mathcal{X}$.

Theorem 1 implies that the proposed mechanism allows individuals to achieve a globally balanced distribution based on local decision-making, meaning that perceptions do not differ by more than one, even if there does not exist an edge between options. Because individuals behave as if they were trying to minimize the opportunity costs across options, the differences in perceptions vanish as the state approaches $\mathcal{X}_u$ (despite the constraints imposed by the interconnected set of options $\mathcal{A}$). It should be highlighted that reaching a globally balanced distribution based on a local interaction mechanism is possible because individuals persistently try to select more attractive alternatives even when both options are perceived as similar (i.e., when the difference in perception between the current and an alternative option is one).

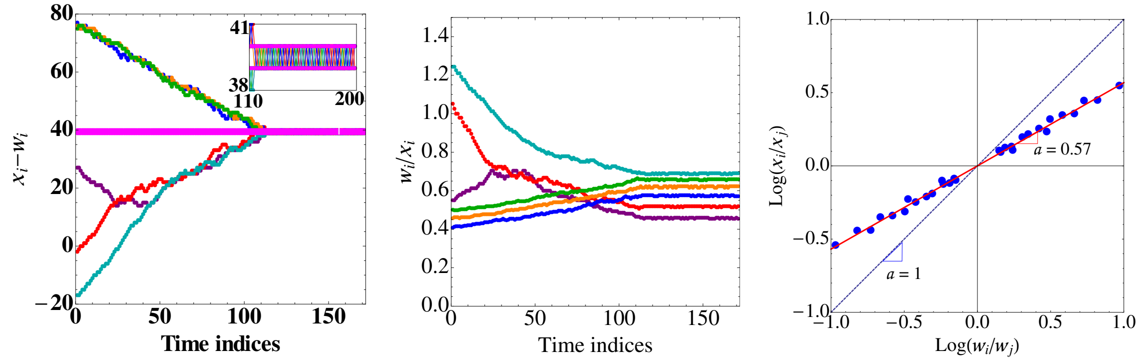
VII. SIMULATIONS

To gain better insight into how group-choice behavior reaches $\mathbf{x} \in \mathcal{X}_u$, consider a low resource scenario where $m = 590$ and $n = 6$. The set $\mathcal{A}$ represents a ring topology and for every option $i \in \mathcal{N}$, the number of resources $w_i = 33, 42, 53, 64, 75, 87$ ($w = 354$ and $r = 0.4$).

Figure 1(a) shows the evolution of the perceptions across options. The distance between the two horizontal lines in the inset plot represents the dynamics within $\mathcal{X}_u$ when $h = 1$ in eq. (3). Due to the particular number of individuals, the level of resource shortage, and the number of available options, group-choice behavior does not converge to a unique state.

Figures 1(b) and 1(c) show that the model captures the tendency of individuals to be less sensitive to resource allocations than predicted by an IFD with strict matching. In particular, Figure 1(b) shows the evolution of the number of resources per individual at each option. Figure 1(c) shows how $\frac{x_i}{x_j}$ relates to $\frac{w_i}{w_j}$ on a logarithmic plot. The dots indicate deviations in group-choice behavior from the diagonal line which represents strict matching. It illustrates that there is low discriminability of resources, represented by $a = 0.57$. When the state reaches $\mathcal{X}_u$ (at around $k = 115$ in Figure 1(a)), variations in the number of resources per individual are small for any option, but the ratio of resources between two options does not match the ratio of individuals choosing these options. Individuals choosing the option with the most resources enjoy higher resources rates.

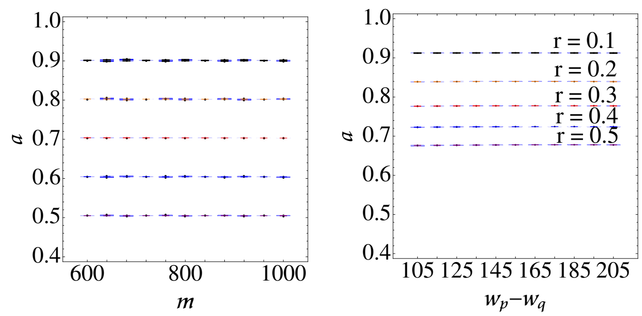
Recall that the value of a (derived in Section III) depends in general on m, r, n, w_p, and w_q. It can be shown that the derivate of eq. (7) with respect to m tends to zero as m tends to infinity. Similarly, the derivative of a with respect to n tends to zero as n tends to infinity. It suggests that the proposed model is not sensitive to such variations. For a finite number of decision-makers, Figure 2(a) quantifies how variations in resource availability affect the degree of matching. In particular, for scenarios with $0.1 \leq r \leq 0.5$, more scarcity leads to a lower degree of matching, but the



(a) (b) (c)

Fig. 1. Dynamics of decision-making leading to the IFD with undermatching in scenarios where eq. (4) represents the perceptions and eq. (9) the local mechanism of decision-making; (a) perceptions associated to each option; (b) number of resources per individual at each option; (c) group-choice ratios when the IFD with undermatching has been achieved.

value does not depend on m. Moreover, note that increasing the maximum difference between resources $w_p - w_q$, as long as Assumption 2 is satisfied, does not affect the degree of matching (see Figure 2(b)).



(a) (b)

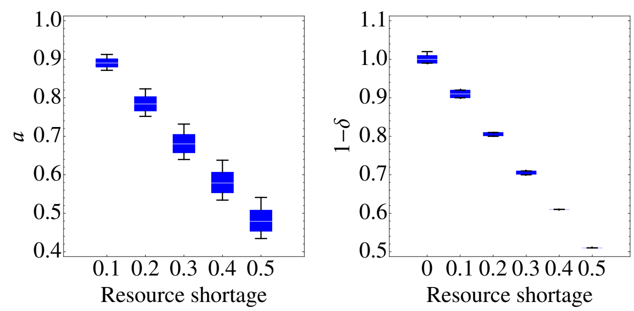
Fig. 2. The effect of resource scarcity on the degree of matching; (a) variation in m; (b) variation in w_p w_q.

Next, to illustrate the impact of misperceptions, consider a scenario where individuals lack perfect information. Similar to the work in [5] we assume that they are well informed about the distribution of resources, but not fully aware of the decisions made by other individuals. Let "noisy" perceptions be represented by

$$\bar{\varphi}_i(x_i, w_i) = c(1 - \delta)x_i - w_i$$

where $(1-\delta)$ is the portion of individuals that are observable. Measures of the degree of matching indicate that noisy perceptions about the number of individuals choosing an option leads to less undermatching, as illustrated in Figure 3(a).

Figure 3(a) shows the value of the degree of matching when we vary the proportion of observable individuals. Because individuals do not perceive the total number of individuals selecting the different options, the ratio of individuals selecting options with a large number of resources is larger than when perfect information about the choices is available. Figure 3(b) shows the proportion of observable individuals that would maximize the profitability for all individuals.



(a) (b)

Fig. 3. Variations in the number of visible individuals; (a) changes in the value of the degree of matching; (b) proportion of visible individuals to achieve an IFD with strict matching.

VIII. Conclusions

The proposed mechanism explains undermatching as a natural outcome resulting from local decision-making. In particular, the degree of matching depends strongly on the level of resource shortage. The dynamics within the invariant set allow that the mechanism to yields a distribution of choices where the perception associated to any two options $i, j \in \mathcal{N}$ does not differ by more than one, despite the constraints imposed on the network of options. Evaluating whether empirical IFD distributions in human group-choice exhibit local or global balanced states is an important direction for future research.

IX. Appendix

Proof: Consider the following Lyapunov candidate function

$$\begin{aligned} \mathcal{V}(\mathbf{x}) :=& \frac{1}{c}\max_i\{\varphi_i(x_i, w_i)\} - \frac{1}{c}\min_i\{\varphi_i(x_i, w_i)\} \\ & - \left\lceil \frac{mr}{n} \right\rceil + \left\lfloor \frac{mr}{n} \right\rfloor \end{aligned} \tag{11}$$

Define $\rho(\mathbf{x}, \mathbf{x}') := \max_i\{|x_i - x_i'|\}$ as the metric on $\mathcal{X}$, $\rho(\mathbf{x}, \mathcal{X}_u) := \inf\{\rho(\mathbf{x}, \bar{\mathbf{x}}) : \bar{\mathbf{x}} \in \mathcal{X}_u\}$ as the distance from $\mathbf{x}$ to $\mathcal{X}_u$, and $B(\mathcal{X}_u; \varepsilon) := \{\mathbf{x} \in \mathcal{X} : 0 < \rho(\mathbf{x}, \mathcal{X}_u) < \varepsilon\}$ as the ε-neighborhood of $\mathcal{X}_u$. Note that, if $\mathbf{x} \in \mathcal{X}_u$ then

$\rho(\mathbf{x}, \mathcal{X}_u) = 0$ and $\mathcal{V}(\mathbf{x}) = 0$. The following four conditions are sufficient to guarantee the stability of $\mathcal{X}_u$.

First, to derive the lower bound on the Lyapunov function $\mathcal{V}(\mathbf{x})$, let $d = \arg\max_i\{i \in \mathcal{N} : |x_i - \bar{x}_i| = \rho(\mathbf{x}, \mathcal{X}_u)\}$ (i.e., option d represents an option (of a possible few) with the largest difference to the closest state in $\mathcal{X}_u$). Let $\varphi_p(x_p, w_p) = \max_i\{\varphi_i(x_i, w_i)\}$ and $\varphi_q(x_q, w_q) = \min_i\{\varphi_i(x_i, w_i)\}$. Then according to eq. (4)

$$\mathcal{V}(\mathbf{x}) = x_p - \frac{w_p}{c} - \left\lceil \frac{mr}{n} \right\rceil - x_q + \frac{w_q}{c} + \left\lfloor \frac{mr}{n} \right\rfloor$$

Moreover, for any $\bar{\mathbf{x}} \in \mathcal{X}_u$ options p and q satisfy eq. (8), and

$$\mathcal{V}(\mathbf{x}) = x_p - \bar{x}_p^u - x_q + \bar{x}_q^l \tag{12}$$

Because $x_i \in \mathbb{N}$, the maximum perception satisfies $cx_p - w_p \geq \lceil \frac{mr}{n} \rceil$ and $x_p - \bar{x}_p^u \geq 0$. Similarly, because the minimum perception satisfies $cx_q - w_q \leq \lfloor \frac{mr}{n} \rfloor$, then $\bar{x}_q^l - x_q \geq 0$. Let $|x_i - \bar{x}_i| = \min\{|x_i - \bar{x}_i^u|, |x_i - \bar{x}_i^l|\}$, such that for any $\mathbf{x} \notin \mathcal{X}_u$ and $\bar{\mathbf{x}} \in \mathcal{X}_u$

$$\mathcal{V}(\mathbf{x}) = |x_p - \bar{x}_p| + |x_q - \bar{x}_q| \tag{13}$$

Consider the following two cases. If $p = d$, we have $|x_d - \bar{x}_d| > |x_q - \bar{x}_q|$. Using eq. (13), $\mathcal{V}(\mathbf{x}) = |x_d - \bar{x}_d| + |x_q - \bar{x}_q|$. Then

$$\mathcal{V}(\mathbf{x}) \geq |x_d - \bar{x}_d| = \rho(\mathbf{x}, \mathcal{X}_u) > \varepsilon_1 = \delta_1$$

If $q = d$, we know that $|x_d - \bar{x}_d| > |x_p - \bar{x}_p|$. Again, $\mathcal{V}(\mathbf{x}) \geq \rho(\mathbf{x}, \mathcal{X}_u) > \varepsilon_1 = \delta_1$. Therefore, for all sufficiently small $\varepsilon_1, 0 < \varepsilon_1 < r$, there exits $\delta_1 = \varepsilon_1 > 0$, such that if $\rho(\mathbf{x}, \mathcal{X}_u) > \varepsilon_1$ then $\mathcal{V}(\mathbf{x}) \geq \varepsilon_1$.

Second, to derive the upper bound on the Lyapunov function $\mathcal{V}(\mathbf{x})$, consider the following two cases. If $p = d$, we have $|x_d - \bar{x}_d| > |x_q - \bar{x}_q|$. Again, using eq. (13), $\mathcal{V}(\mathbf{x}) = |x_d - \bar{x}_d| + |x_q - \bar{x}_q|$. Then

$$\mathcal{V}(\mathbf{x}) \leq 2|x_d - \bar{x}_d| = 2\rho(\mathbf{x}, \mathcal{X}_u) < 2\delta_2 = \varepsilon_2$$

If $q = d$ we know that $|x_d - \bar{x}_d| > |x_p - \bar{x}_p|$ and $\mathcal{V}(\mathbf{x}) \leq 2\rho(\mathbf{x}, \mathcal{X}_u) < 2\delta_2 = \varepsilon_2$. Therefore, for all sufficiently small $\varepsilon_2 > 0$, there exits $\delta_2 = \frac{\varepsilon_2}{2} > 0$, such that if $\rho(\mathbf{x}, \mathcal{X}_u) < \frac{\varepsilon_2}{2}$ then $\mathcal{V}(\mathbf{x}) \leq \varepsilon_2$.

Third, we need to show that for any $\mathbf{x}(0) \in B(\mathcal{X}_u; \varepsilon)$, any sequence of events in the set $\mathcal{E}$ yields

$$\mathcal{V}(\mathbf{x}(k)) \geq \mathcal{V}(\mathbf{x}(k+1))$$

Let $\mathcal{V}_1(\mathbf{x}) = \max_i\{\varphi_i(x_i, w_i)\} - \lceil \frac{mr}{n} \rceil$ and that $\mathcal{V}_2(\mathbf{x}) = \lfloor \frac{mr}{n} \rfloor - \min_i\{\varphi_i(x_i, w_i)\}$, so that $\mathcal{V}(\mathbf{x}) = \mathcal{V}_1(\mathbf{x}) + \mathcal{V}_2(\mathbf{x})$. Assume that $\mathbf{x}(k) \notin \mathcal{X}_u$ and let option p be an option that has the largest perception, for which individuals have at least one locally available attractive alternative (i.e., with a lower opportunity cost). We want to show that $\varphi_p(x_p(k), w_p) > \varphi_j(x_j(k+1), w_j)$ for $p \in \mathcal{M}_j$ for some alternative option j. Note that because $\varphi_p(x_p(k), w_p) > \varphi_j(x_j(k), w_j)$, we know that $\varphi_p(x_p(k), w_p) - \varphi_j(x_j(k), w_j) \geq 1$. Then

$$\begin{aligned}\varphi_p(x_p(k), w_p) &\geq \varphi_j(x_j(k), w_j) + 1 \\ &= \varphi_j(x_j(k) + \mathbb{1}_j^+, w_j) = \varphi_j(x_j(k+1), w_j)\end{aligned}$$

Thus, $\mathcal{V}_1(\mathbf{x}(k)) \geq \mathcal{V}_1(\mathbf{x}(k+1))$.

Next, to show that $\mathcal{V}_2$ is non-increasing suppose that $\mathbf{x}(k) \notin \mathcal{X}_u$ and that option q has the lowest perception, with at least one less attractive locally available alternative (i.e., with a higher opportunity cost). We know that $\varphi_q(x_q(k), w_q) < \varphi_i(x_i(k), w_i)$ and $\varphi_i(x_i(k), w_i) - \varphi_q(x_q(k), w_) \geq 1$ for some option i. Then

$$\begin{aligned}\varphi_q(x_q(k), w_q) &\leq \varphi_i(x_i(k), w_i) - 1 \\ &= \varphi_i(x_i(k) - \mathbb{1}_i^-, w_i) = \varphi_i(x_i(k+1), w_i)\end{aligned}$$

Thus, $\mathcal{V}_2(\mathbf{x}(k)) \geq \mathcal{V}_2(\mathbf{x}(k+1))$, which guarantees that $\mathcal{V}(\mathbf{x}(k)) \geq \mathcal{V}(\mathbf{x}(k+1))$.

Finally, to show that $\mathcal{V}(\mathbf{x}(k)) \to 0$ as $k \to \infty$, let $\tau = \inf\{\mathcal{V}(\mathbf{x}(k)) : k \in \mathbb{N}\}$. Using eq. (13) we have that $|x_p(k) - \bar{x}_p| + |x_q(k) - \bar{x}_q| = \tau$ which implies that $x_p(k)$ is above the equilibrium ($x_p(k) > \bar{x}_p$), $x_q(k)$ is below the equilibrium ($x_q(k) < \bar{x}_q$), or both. In other words, option p is not an attractive option, and some individuals will change for an alternative. Similarly, if option q is the most attractive option then some individuals will change their current option for option q. Because $\mathcal{V}$ is a non-increasing function, then

$$\mathcal{V}(\mathbf{x}(k+1)) < \tau = \mathcal{V}(\mathbf{x}(k))$$

which contradicts the fact that τ is the infimum of $\mathcal{V}(\mathbf{x}(k))$. Thus, $\mathcal{V}(\mathbf{x}(k)) \to 0$ as $k \to \infty$. ■

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